VSB — TECHNICAL UNIVERSITY OF OSTRAVA

FACULTY OF ECONOMICS

DEPARTMENT OF FINANCE

Optimalizace akciového portfolia

Optimization of Stock Portfolio

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Ostrava 2019

VŠB - Technical University of Ostrava
Faculty of Economics
Department of Finance

Diploma Thesis Assignment

Student: **Bc. Linjing Wu**

Study Programme: N6202 Economic Policy and Administration

Study Branch: 6202T010 Finance

Title: Optimization of Stock Portfolio
Optimalizace akciového portfolia

The thesis language: English

Description:

1. Introduction
2. Description of Portfolio Optimization
3. Description of Portfolio Backtesting and Performance Measures
4. Application of Portfolio Optimization
5. Conclusion

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List of Abbreviations
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Extent and terms of a thesis are specified in directions for its elaboration that are opened to the public on the web sites of the faculty.

Supervisor: **doc. Ing. Aleš Kresta, Ph.D.**

Date of issue: 23.11.2018
Date of submission: 26.04.2019

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The declaration

"I hereby declare that I have elaborated the entire thesis including annexes myself."

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Contents

1. Introduction

Portfolio optimization is the process of comparing and selecting the best portfolio from all of the portfolios that are alternative. Prior to the introduction of Markowitz's portfolio theory, the concept of diversification already existed. But in 1952, the portfolio theory was proposed. The American economist Markowitz proposed the Portfolio Theory in 1952 in his paper "Portfolio Selection". He proposed that expected return, and variance of return of the portfolio should be the criteria for portfolio selection.

The goal of the diploma thesis is to compare the out-of-sample performance of portfolio allocation strategies we select. The selected strategies are Naive strategy, Markowitz model, minimum variance portfolio and Tobin model. Since the topic is about optimization of stock portfolio, the data we select are the adjusted stock prices of 30 stocks of Hang Seng Index. The criteria of selecting the stock are size of market capitalization and data availability. The data are weekly data between January 4th 2009 and December 30th 2018.

The thesis is divided into 5 parts. The first and last parts are introduction and conclusion. The second part is the description of portfolio optimization. In chapter 2, we introduce the portfolio theory, the historical data approach, the strategies we choose, and expected utility function which helps us to find the optimal portfolio. Chapter 3 is the description of portfolio backtesting and portfolio measures. We describe the backtesting framework and performance measures we choose, which are maximum drawdown, Sharpe ratio and Jensen's alpha.

In chapter 4, we divide our data into in-sample period and out-of-sample period. We apply strategies separately in these two periods, and backtest the strategies in out-of-sample period. Then we compare each strategy by performance measures and determine which one is the best.

2. Description of Portfolio Optimization

A group of assets is called a portfolio. If there are N assets in the economy, then we can define the portfolio through the N vector, which includes the proportion of wealth invested in each N asset.

According to investment objectives, portfolio optimization is the process of comparing and selecting the best portfolio from all of the portfolios that are under consideration. Typically, the objectives of investing can be maximization of factors such as expected returns and minimization of factors such as financial risk. Portfolio selection and portfolio management are the most important problems from the past that have attracted the attention of investors. To solve these problems, economist Harry Markowitz proposed his model that was named Markowitz or mean-variance model. In this chapter, we are going to introduce the portfolio theory, estimation of historical data, assets allocation strategies we are going to use, and expected utility function which can help us solve the problem about optimal portfolio.

The description of chapter 2 is based on the book: Markowitz (2009) and Kresta (2015).

2.1 Portfolio Theory

Portfolio theory can be divided as narrow and generalized. The narrow sense of portfolio theory refers to Markowitz's portfolio theory. And the generalized portfolio theory also includes the Capital Asset Pricing Model and the Modern Market Theory, in addition to the classic portfolio theory and the various alternative portfolio theories. Because traditional Efficient Markets Hypothesis (EMH) can't explain market anomalies, portfolio theory is challenged by Behavioral Finance Theory.

When talking about portfolio theory, the most famous person that appears in the minds of most people is Harry Max Markowitz (born August 24, 1927), an American economist. In this sub chapter, we are going to introduce the early portfolio theory and the famous modern portfolio theory.

2.1.1 Early History of Portfolio Theory

This sub-chapter is based on Markowitz (2009). Prior to the introduction of Markowitz's portfolio theory, the concept of diversification already existed. In 1935, Hicks proposed the "Separation Theorem" and explained the demand of money as the consequence of investors' expectations of high returns and low risks. At the same time, he believed that, theory of money should be constructed. He introduced risk into his analysis. Specifically, he noted "The risk-factor comes into our problem in two ways: First, as affecting the expected period of investment, and second, as affecting the expected net yield of investment". But there are also some shortcomings of his research. He never specified standard deviation or any other measures as the measure of risk. Thus, he was unable to establish formulas about the relationship between risk on the portfolio and risk on individual assets. And there is no distinguish between efficient and inefficient portfolios, also no drawing of an efficient frontier.

Kenes in 1936 and Hicks in 1939 proposed the concept of risk compensation. It was believed that because of the existence of uncertainty, certain financial products should be supplemented with certain risk compensation in addition to interest rates. Hicks also proposed the problem of asset selection, and he thought risk can be diversified.

Marschak in1938 made some efforts to construct an ordinal theory of choice under uncertainty and also noticed that people always like high mean and low standard deviation. But his theory was not a step toward portfolio theory because it did not consider the portfolio. The parameters such as mean, standard deviation, correlation were used directly in the utility and transformation functions. And there was no analyzing about how they fit together to form the investor's entire portfolio. On the other hand, Marschak's 1938 work is a milestone to a market theory, and was developed later in Tobin's theory and CAPMs. This was the biggest advancement in economics under risk and uncertainty before publication of von Neumann and Morgenstern in 1944.

Williams in1938 proposed the Dividend Discount Model. In his book, he observed that the future dividends of the stock or the interest and principal of a bond may be uncertain. He said that, in this case, probabilities should be assigned to various possible values of the security and the mean of these values used as the value of the security. Finally, he believed that by investing in enough securities, risk can be eliminated and that there was always a combination that satisfied the maximization of return and minimization of risk.

Leavens in 1945 illustrated the benefits of diversification on the assumption that risks are independent. Von Neumann in 1947 then applied the concept of expected utility to propose a decision-making method under uncertainty conditions.

Tobin in 1958 assumed that investors were seeking a mean–variance-efficient combination of monetary assets. Tobin presented his groundbreaking results, now known as Tobin separation theorem. He proposed the assumption of a portfolio selection model with *N* risky assets and one risk-less asset, cash. Since these assets were monetary assets, the risk should be market risk, not default risk.

2.1.2 Modern Portfolio Theory

The American economist Markowitz proposed the Portfolio Theory in 1952 in his paper “Portfolio Selection”, for which he won the Nobel Prize in Economics. And he is often called the father of modern portfolio theory. Through the analysis of mean-variance model, he got the conclusion that the risk can be effectively reduced through the portfolio.

The theory contains two important elements: the mean-variance analysis and the efficient frontier. The basic ideas of Markowitz's portfolio theory are: (1) investors determine the appropriate assets in the portfolio; (2) calculate and analyze the expected returns and risks of these assets during the holding period; (3) establish alternative efficient set; (4) combined with specific investment objectives, determine the final optimal portfolio.

The core point of Markowitz's portfolio theory is that investors' investment aspirations are to pursue high expected returns and to avoid risks as much as possible.

Therefore, for a portfolio of securities, investors should attach importance not only to the expected benefits, but also to the risks related. The theory is based on the following assumptions:

1. The analysis is based on single period investment model. Therefore, the change of portfolio structure during the investment period is not allowed.
2. Investors are risk averse and rational and the investor's utility function is concave and increasing.
3. The risk of the portfolio is based on the variability of returns of the portfolio.
4. Infinitely small amount of money can be invested into the particular assets.
5. We assume only expected return and variance as the portfolio parameters.
6. The securities market is efficient. That is, the changes in the risk and return of each type of securities in the market are available for investors. And there is no transaction costs or taxes.
7. The returns between each type of security are related, that is, the correlation coefficient between any two securities can be known by calculation, so that the all available portfolios can be found.
8. The higher the investment return, the greater the investment risk; the lower the investment income, the smaller the investment risk.

Modern portfolio theory identifies two aspects of the investment problem. First, an investor wants to maximize the expected rate of return on the portfolio. Second, an investor wants to minimize the risk of the portfolio. The goal of these two aspects is to maximize the expected rate of return for any given, acceptable level of risk. Alternatively the goal can be stated as: minimize the risk for any given, acceptable level of expected return. In this theory, risk is associated with the variance, or more commonly, the standard deviation of the portfolio.

2.2 Historical Data Approach

Parameters of portfolio are very important in the process of optimization. In this section, we are going to introduce the calculation and estimation of returns and other parameters. It is very important to estimate the expected returns and covariance of the individual assets because they also determine the composition of optimal portfolio.

Return refers to the gain or loss of a security in a specific period. The return consists of the income and the capital gains relative to an investment. It's usually quoted as percentage. Alternatively, we can define return as the relative change of individual assets' prices or the relative change of the portfolio value.

2.2.1 Individual Assets Returns and Estimation

We assume that there are N assets in the portfolio, and the size of time series is m. Then we can get a matrix. Since the returns can be discrete and continuously compounding, the calculation of individual assets returns can be different. Discrete returns can be calculated as follows,

$$R_t = \frac{P_t - P_{t-1} + D}{P_{t-1}} = \frac{P_t + D}{P_{t-1}} - 1 , \tag{2.1}$$

where R_t represents the return at time t, P_t represents the price of the assets at time t, P_{t-1} is the price of assets at time $t-1$.

For continuously compounding returns, it's assumed that the time interval of price changes is infinitely small, and it can be calculated as follows,

$$R_t = \ln \frac{P_t + D}{P_{t-1}} = \ln(P_t + D) - \ln P_{t-1} . \tag{2.2}$$

The expected rate of return, refers to the future achievable rate of return of an asset under uncertain conditions. Based on the historical observations of returns, we can estimate the population mean as,

$$E(R_i) = \frac{1}{m} \sum_{t=1}^{m} R_{i,t} , \tag{2.3}$$

where m is the number of returns.

Then we can calculate the variance and standard deviation of the returns as follows,

$$\sigma_i^2 = \frac{1}{m-1}\sum_{t=1}^{m}[R_{i,t} - E(R_i)]^2 \quad , \tag{2.4}$$

$$\sigma_i = \sqrt{\sigma_i^2} = \sqrt{\frac{1}{m-1}\sum_{t=1}^{m}[R_{i,t} - E(R_i)]^2} \, , \tag{2.5}$$

Covariance is a measure of the joint variability of two random variables. Variance is a special kind of covariance when there is only one random variable. If the two variables change in the same direction, that is, if one of them is greater (or less) than its own expected value and the other is greater (or less) than its own expected value too, the covariance between the two variables is positive. On the contrary, the covariance between the two variables is a negative value. The covariance of returns can be calculated as follows,

$$\sigma_{i,j} = \frac{1}{m-1}\sum_{t=1}^{m}[R_{i,t} - E(R_i)]\cdot[R_{j,t} - E(R_j)] \, , \tag{2.6}$$

and the correlation of returns can be calculated as follows,

$$\rho_{i,j} = \frac{\sigma_{i,j}}{\sigma_i \cdot \sigma_j} \, . \tag{2.7}$$

2.2.2 Portfolio Return

After calculating the returns of individual assets, we consider the returns of portfolio. Portfolio return refers to the gain or loss realized by investing in portfolios. Since the portfolio is composed of different individual assets, we can calculate portfolio parameters based on the individual assets. And the formula of value of portfolio is as follows,

$$P_{P,t} = \sum_{i=1}^{N} P_{i,t} \cdot v_i \, , \tag{2.8}$$

where $P_{i,t}$ is the particular assets' prices, and v_i represents the corresponding quantities of these assets.

Same as the individual assets' returns, the portfolio returns can also be calculated under discrete condition and continuously compounding condition. And the discrete return of the portfolio can be calculated as follows,

$$R_{P,t} = \frac{P_{P,t} - P_{P,t-1}}{P_{P,t-1}} = \frac{\sum_{i=1}^{N} P_{i,t-1}(1+R_{i,t})v_i - \sum_{i=1}^{N} P_{i,t-1}v_i}{\sum_{i=1}^{N} P_{i,t-1}v_i}, \tag{2.9}$$

and the continuously compounded return can be calculated as,

$$R_{P,t} = \ln P_{P,t} - \ln P_{P,t-1} = \ln(\sum_{i=1}^{N} P_{i,t-1}e^{r_{i,t}}v_i) - \ln(\sum_{i=1}^{N} P_{i,t-1}v_i). \tag{2.10}$$

To simplify the above complicated formulas, we introduce weight to the formula. The calculation of weight is as follows,

$$w_i = \frac{P_{i,t-1}v_i}{\sum_{i=1}^{N} P_{i,t-1}v_i}, \tag{2.11}$$

which means that the weight of i-th asset is the percentage of amount that is invested in the i-th asset to the total invested amount. Then the formula of returns can be simplified, for discrete return of the portfolio:

$$R_{P,t} = \sum_{i=1}^{N} w_i \cdot R_{i,t}, \tag{2.12}$$

and for continuously compounded return:

$$R_{P,t} = \ln \sum_{i=1}^{N} w_i \cdot e^{r_{i,t}}. \tag{2.13}$$

The estimation of expected return of portfolio and it's variance and standard deviation are introduced in the next section as the important part of Markowitz's mean-variance analysis.

2.3 Markowitz's Mean-Variance Analysis

Markowitz proposed that expected return, and variance of return of the portfolio should be the criteria for portfolio selection. According to the second assumption of Modern Portfolio Theory, investors are risk averse. That is to say, they do not like the risk. If they need to afford more risk, they must be compensated by higher expected returns. For two portfolios with same conditions except risk, they will choose the one with less risk.

To conduct the mean-variance analysis, we need to define the calculation of expected return and variance (also the standard deviation). The expected return on the portfolio is a weighted average of the expected returns on individual assets. We assume that there are *N* assets included in the portfolio, and we can estimate the expected returns of particular asset $E(R) = \{E(R_1, ..., E(R_N)\}^T$. Assuming the portfolio composition $x = \{x_1, ..., x_N\}^T$, then we can calculate the portfolio expected return $E(R_P)$ as:

$$E(R_P) = \sum_{i=1}^{N} x_i \cdot E(R_i) = x^T \cdot E(R), \tag{2.14}$$

where R_P is the return on the portfolio, R_i is the return on asset i and x_i is the weighting of *i*-th asset in the portfolio. Here, x_i is the same as w_i in formula (2.11).

Assuming the covariance matrix of returns $Q = \{\sigma_{i,j}, i = 1,..., N, j = 1,..., N\}$, we can compute the portfolio variance σ_P^2 and standard deviation σ_P as follows:

$$\sigma_P^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} x_i \cdot \sigma_{i,j} \cdot x_j = x^T \cdot Q \cdot x \quad , \tag{2.15}$$

$$\sigma_P = \sqrt{\sigma_P^2} \quad , \tag{2.16}$$

where $\sigma_{i,j} = \sigma_i \cdot \sigma_j \cdot \rho_{i,j}$ is the estimated covariance of the periodic returns on the two assets, or alternatively denoted as $\sigma(i, j)$, cov_{ij} or *cov(i,j)*.

2.3.1 Efficient Set of Markowitz Model

Efficient set, also known as the efficient frontier, was originally developed by Markowitz as a method of portfolio selection. Markowitz (1952) explained that "since there were two criteria, risk and return, it was natural to assume that investors selected from the set of Pareto optimal risk-return combinations." The optimal risk-return combination of a portfolio lies on an efficient frontier of maximum returns for a given level of risk based on mean-variance portfolio construction.

Assume that an investor can invest only into the risky assets and no short selling is allowed. According to the requirements of constructing the efficient frontier, we have to formulate three types of problems and there are three distinct steps. In the first step, we need to find the portfolio with the minimal standard deviation. In the second step, we need to find the portfolio with the maximal expected return. In the last step, we need to select the portfolios for interior points of the efficient set. They are expressed in the following text.

In the first step, the portfolio with minimum risk can be found by solving following problem,

$$\begin{cases} \min \sigma_P^2 \\ \sum_{i=1}^{N} x_i = 1 \\ x_i \geq 0, i = 1,\dots,N \end{cases} \tag{2.17}$$

In the second step, the portfolio with maximum return can be found by solving the second problem,

$$\begin{cases} \max E(R_P) \\ \sum_{i=1}^{N} x_i = 1 \\ x_i \geq 0, i = 1,\dots,N \end{cases} \tag{2.18}$$

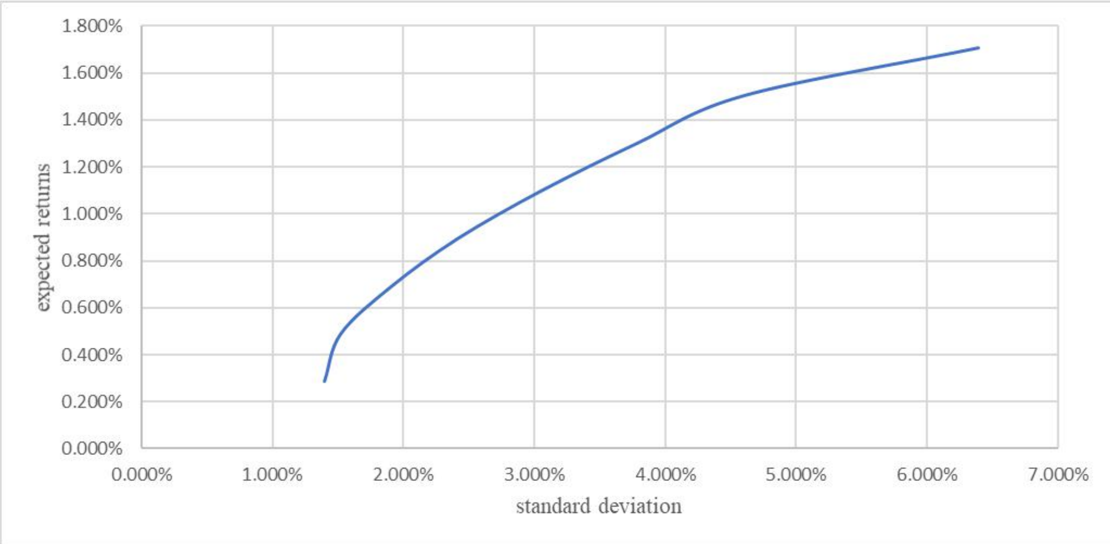
In the last step, for the interior points of the efficient set, the problem to be solved is as follows,

$$\begin{cases} \min \sigma_P^2 \\ E(R_P) \geq R_{j,generated} \\ \sum_{i=1}^{N} x_i = 1 \\ x_i \geq 0, i = 1,\dots,N \\ R_{j,generated} = R_{P,\min} + j \cdot \dfrac{(R_{P,\max} - R_{P,\min})}{100}, j = 1,\dots,99 \end{cases} \tag{2.19}$$

where $R_{p,min}$ represents the return of the minimum-variance portfolio and $R_{P,max}$ represents the return of the maximum-return portfolio. And $R_{j,generated}$ represents the

j-th predefined value computed in equidistant intervals between $R_{p,min}$ and $R_{P,max}$.

Chart 2.1 Efficient set



source: own elaboration

Chart 2.1 represents the Markowitz Efficient Set. As we can see, the expected return of portfolio is represented on the Y-axis and risk (standard deviation) is represented on the X-axis. Feasible set is the set of all alternative portfolio that we can choose, and the efficient set is the upper boundary line of the feasible set. The efficient set is an upward curve, thus the relationship between risk and expected return of portfolio is positive.

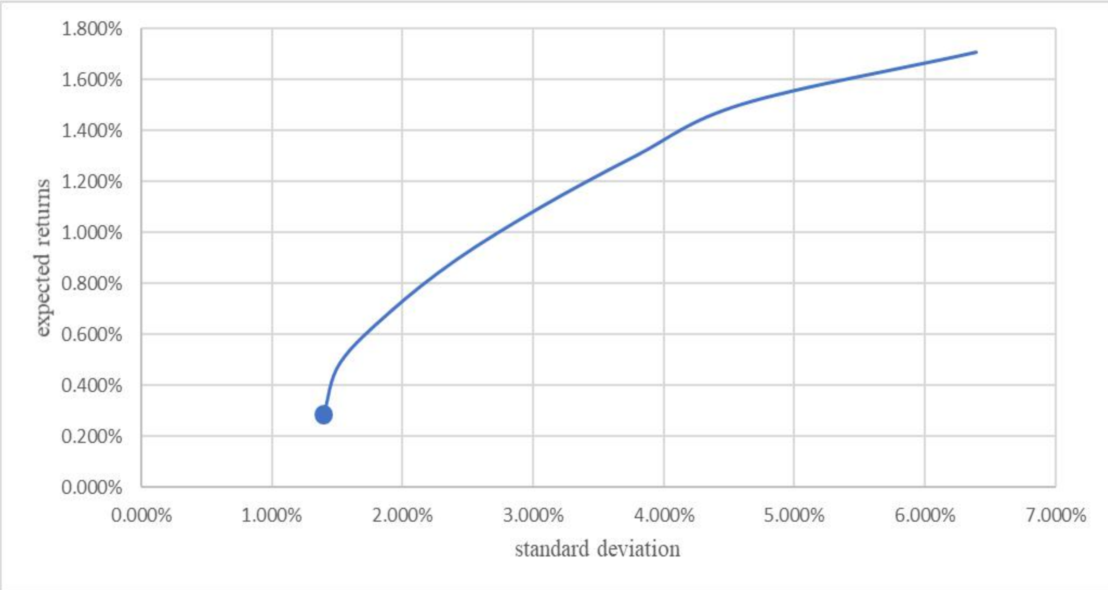
2.4 Minimum Variance Portfolio

For risk-averse investors, the minimum variance portfolio is a well-diversified portfolio with the lowest risk among alternative portfolios. It's a kind of risk-based approach, because the main factor that is considered is risk. It is different from the Markowitz model which consider both risk and expected return.

Due to the relationship between risk and return on the efficient set, the return of this investment method is also the lowest. The portfolio can be found by solving following problem,

$$\begin{cases} \min x^T \cdot Q \cdot x \\ \sum_{i=1}^{N} x_i = 1 \\ x_i \geq 0, i = 1, \ldots, N \end{cases} \tag{2.20}$$

Chart 2.2 Minimum variance portfolio



Source: own elaboration

In the above chart, it shows the minimum variance frontier, and the red pint is the portfolio with minimum risk (represented by standard deviation).

2.5 Portfolio with Different Conditions

In Markowitz model, we assume that the short selling as well as risk-free investment is not allowed. It means that the investors are only allowed to invest into risky assets in the market. As we change the two conditions, we can get more possibilities：

1) short selling is allowed while risk-free asset is allowed too.

2) Short selling is allowed but risk-free investment is not allowed, in this case, the Black's model is a typical example. Similar to the efficient frontier construction in Markowitz model, we only need to change the constraint of x_i as follows,

$$x_i \geq -1, \text{ for } i = 1, 2, \ldots, N. \tag{2.21}$$

3) Short selling is not allowed but risk-free investment is allowed, in this case, the Tobin model can be a representative. We need to solve two kind of problems.

First one is market portfolio. Market portfolio is the special case of the tangency portfolio. It consists of all risky assets which are available at the market. The portfolio is selected with the aim of having a maximal slope of the CML line. The objective function and constraints can be written as follows,

$$\begin{cases} \max \dfrac{E(R_M) - R_F}{\sigma_M} \\ x_F + \sum_i x_i = 1 \\ x_i \geq 0, \text{for } i = 1,2,...N \\ x_F = 0 \end{cases} \tag{2.22}$$

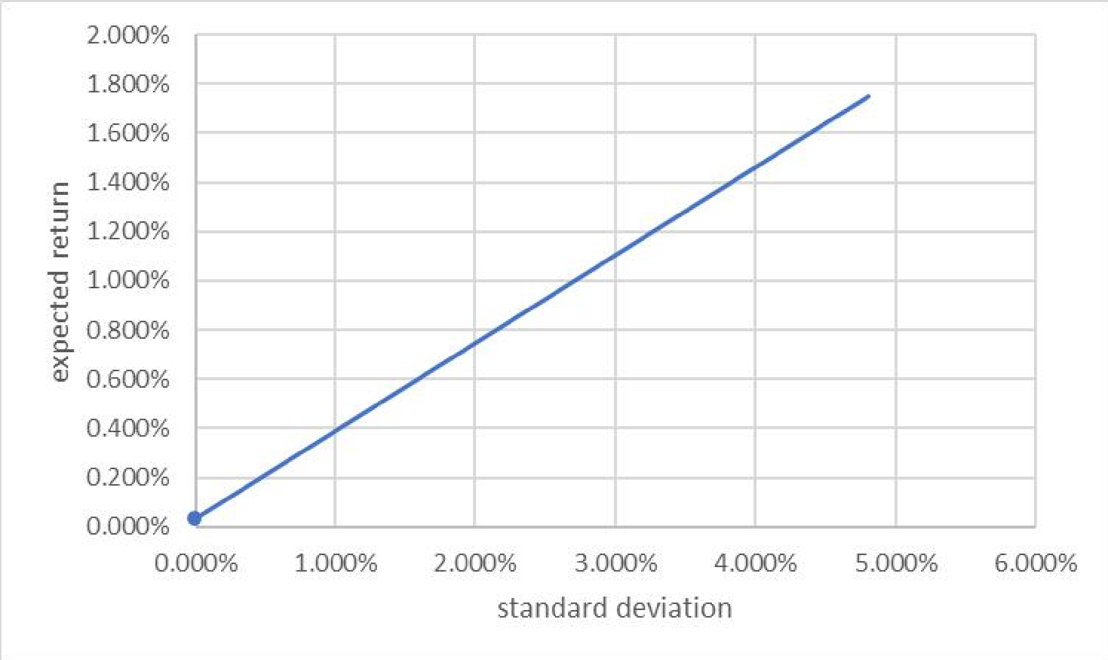
Next one is efficient portfolios. The objective function and constraints are as follows,

$$\begin{cases} E(R_P) \to \max \\ x_F + \sum_i x_i = 1 \\ x_i \geq 0, \text{for } k = 1,\ 2,\ ...,\ N \\ -\infty \leq x_F \leq \infty \\ \sigma_P = \sigma_{P-gen} \end{cases} \tag{2.23}$$

4) Short selling is not allowed while risk-free investment is not allowed either, in this case, we can construct the Markowitz efficient frontier as introduced in previous chapter.

As another strategy, we apply the portfolio with risk-free investment. In theory, the risk-free rate is the minimum return an investor can get for any investment because he bears no risk. But in reality, risk-free doesn't exist because even the safest investments will bring a very small risk. Thus, we often apply the interest rate on government bond as the risk-free rate. And the efficient frontier can be constructed similar as the Markowitz model in previous chapter, but we need to add risk-free asset into the portfolio. And the efficient frontier with risk-free asset is shown in chart 2.3.

Chart 2.3 Efficient frontier with risk-free asset



Source: own elaboration

2.6 Naive Strategy

The naive strategy also refers to *1/N* portfolio strategy. It's simple and easy to implement compared to Markowitz model. In Markowitz model, investors are allowed to construct portfolios by investing in large number of securities. And it makes better performance than investment in single asset. But it's complicated and sensitive to small changes. Thus we introduce the naive strategy which is easier and can also achieve the goal of diversification.

Instead of constructing a portfolio by applying complicated approach, the investors can establish an optimal portfolio by investing in large number of assets equally. And the portfolio is so called as *1/N* portfolio.

Same to Markowitz model, the idea of naive strategy is that investing in large number of securities can reduce the portfolio risk. If the number of securities is large enough, the return of single security will not affect the whole portfolio. But the difference is the weights of stocks. We need to estimate weights of stocks in Markowitz model, but in naive strategy, we apply same weights for each stock. The advantages of naive strategy are: 1) there are no parameters to estimate, thus it's simple; 2) the strategy is not affected by the estimation error.

2.7 Maximization of Utility Function

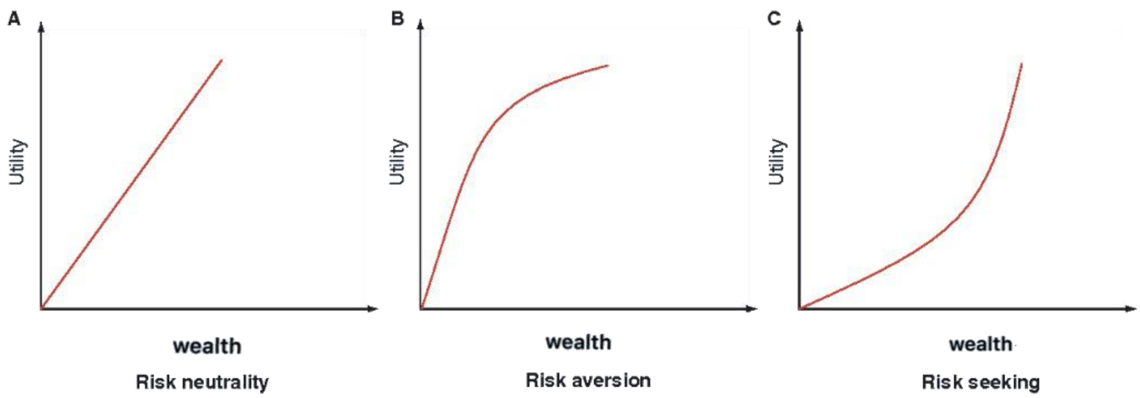
Selecting the optimal portfolio of financial assets is one of the basic problems in financial modelling. The choice of the optimal portfolio depends on the risk profile of the investor, but even the degree of the risk aversion is different. To solve the problem about optimal portfolio, we introduce the expected utility function.

For an investor who is maximizing the expected return from the portfolio while minimizing the risk, the utility function can be assumed as follows,

$$U = E(R_P) - k \cdot \sigma_P^2 \quad , \tag{2.24}$$

where k is the attitude to risk. From the above formula, we can find that every unit increase in variance σ_P^2 need to be compensated by k-times increase in expected return $E(R_P)$ in order to keep the same value of utility U. As the parameter k represents the risk profile of investor, the categories of investors can be divided by different value intervals of k: 1) k>0, risk averse investors, 2) k=0, risk neutral investor, 3) k<0, risk lovers.

Chart 2.4 Risk attitude



Source: own elaboration

Under the assumption of rational investor, the investor seeks for the highest utility. Then we can find the value by solving the following problem,

$$\begin{cases} \max E(R_P) - k \cdot \sigma_P^2 \\ \sum_{i=1}^{N} x_i = 1 \\ x_i \geq 0, i = 1, \ldots, N \end{cases} \tag{2.25}$$

As we assumed before in Modern Portfolio Theory, the investor is risk averse, and we can discuss about the circumstances when $k \geqslant 0$:

1) k=0, $\max E(R_P) - k \cdot \sigma_P^2 = \max E(R_P)$, the optimal portfolio is the maximum return portfolio,

2) $k \rightarrow \infty$, the optimal portfolio is approaching the minimum-variance portfolio,

3) $k \in (0,\infty)$, with the knowledge of particular person's risk aversion level, we can obtain different points of optimal portfolios on the efficient set. And by increasing the value of k, the portfolio becomes more diversified and the values of expected return and standard deviation both decrease.

3. Description of Portfolio Backtesting and Performance Measures

In the previous chapter, we describe the strategies of portfolio optimization, we proposed the optimization problem which can be applied to find the optimal portfolio. In this chapter, we are going to introduce portfolio backtesting and performance measures that can evaluate if the strategies performed well.

Backtesting is a general method for seeing how well a strategy or model would have performed ex-post. Backtesting assess the feasibility of strategies by discovering how it would play out using historical data. If the backtesting is valid, traders and analysts may be confident to continue using it.

Performance measures are quantifiable indicators used to assess how well strategies are achieving its desired objectives. There are a lot of performance ratios that have been developed in the past. And the selected measures that we are going to introduce are Maximum drawdown (MDD) and Sharpe Ratio and Jensen's alpha.

The theory of backtesting and performance measures are used to trading strategies, and in this chapter, we apply them to investment strategies.

3.1 Backtesting Framework

Backtesting should reflect the actual investment environment as much as possible, including investment objectives and trading environment.

Backtesting allows traders to use historical data to simulate investment strategies to produce results and analyze risk and profitability, before taking any risk of real capital. Through backtesting procedure, we can get the answer of what would happen if we followed each strategy in the history.

In the backtesting procedure, we calculate the portfolio composition for each observation. At the time t, the weights of the portfolio are determined by the prices of the assets in historical window ($t-m$, $t-1$), where m stands for the size of historical data, and variable t is the start date of backtesting, historical window is the interval of past observations used for estimation.

We calculate the ex-post portfolio returns as follows,

$$R_{P,t} = R_t \cdot w_t \,, \tag{3.1}$$

where R_t represents the ex-post observed asset returns and w_i represents the weights of assets in portfolio, which is obtained by portfolio optimization based on the prices of assets in historical window (*t-m,t-1*).

Also we can calculate the ex-post wealth (portfolio value) evolution as follows,

$$W_t = W_{t-1} \cdot (1 + R_{P,t}) \,, \tag{3.2}$$

where W_t represents the accumulated wealth after time t.

3.2 Performance Measures

The primary goal of assessing the portfolio performance is to measure the value creation provided by the portfolio management industry. Many investors mistakenly think that their success on their portfolios are based on returns alone. Only few investors consider the related risks while achieving returns. It's important to know how to quantify and measure risk with the variability of returns. Nowadays, the investment in mutual and exchange-traded funds has spread among small investors in the world, thus there exists increased demand for reliable performance indicators.

Through the backtesting procedure, we know how to get the ex-post portfolio returns and ex-post wealth evolution. Then we can use them to measure the performance of each strategies. We are going to introduce indicators that we are going to apply.

3.2.1 Maximum Drawdown

The drawdown is the measure of the decline from a historical peak in some variable. The maximum drawdown (MDD) up to time T is the maximum of the Drawdown over the history of the variable. Alternatively, it can be defined as the largest drop from a peak to a bottom of a portfolio value. And it is an indicator of downside risk over a specified time period. The maximum drawdown is expressed in percentage terms and computed as:

$$MDD = \frac{\textit{(Peak value-Trough value)}}{\textit{Peak value}}. \tag{3.3}$$

Similar to the backtesting procedure, we assume the wealth path as W_t, we can measure the decline from the historical peak at time *i*, the drawdown (DD) can be calculated as follows,

$$DD_i = \frac{\max\limits_{t\in(0,i)} W_t - W_i}{\max\limits_{t\in(0,i)} W_t} = 1 - \frac{W_i}{\max\limits_{t\in(0,i)} W_t} \quad , \tag{3.4}$$

where W_i is the portfolio value at time *i*, and $\max\limits_{t\in(0,i)} W_t$ is the historical peak before time *i*. The above formula represents the size of the decline the investor will suffer at time *i* related to the historical maximal peak. And we can measure the maximum drawdown over the period *(0,T)*, the formula is as follows:

$$MDD_{0,T} = \max_{i\in(0,T)} \left(1 - \frac{W_i}{\max\limits_{t\in(0,i)} W_t}\right) \quad . \tag{3.5}$$

Since the MDD measures the largest peak-to-trough decline in the value of a portfolio, it only measures the size of the largest loss, without taking into consideration the frequency of large losses and it does not indicate how long it took an investor to recover from the loss, or if the investment even recovered at all.

3.2.2 Sharpe Ratio

The Sharpe ratio, also known as the Sharpe index, is a standardized indicator of portfolio performance evaluation. Study on Sharpe ratio shows that the size of risk plays a fundamental role in determining the performance of the portfolio. The risk-adjusted rate of return is a comprehensive indicator that considers both returns and risks, with a view to eliminating the negative impact of risk factors on performance evaluation. The Sharpe ratio is one of the three classic indicators (which are Sharpe ratio, Jensen's alpha and Treynor ratio) that can simultaneously consider the returns and risks. There is a conventional feature in investment, that is, the higher the expected return on the investment target, the higher the risk that the investor can bear; on the contrary, the lower the expected return, the lower the risk of volatility. Therefore, the main purpose of rational investors to choose investment targets and portfolios is to pursue the greatest rewards under fixed acceptable risks, or to pursue the lowest risks under fixed expected returns.

The core idea of Sharpe is that rational investors will choose and hold an efficient portfolio. He believes that while building a risky portfolio, the investor should at least require a return on investment to achieve a return on risk-free investment, or more.

From mathematical view, the Sharpe ratio is the ratio between the excess expected return and its volatility, and the formula according to Sharpe (1966) is as follows:

$$SR_{R,R_{RF}} = \frac{E(R - R_{RF})}{\sigma_{R-R_{RF}}} = \frac{E(R) - R_{RF}}{\sigma_R}, \tag{3.6}$$

where R_{RF} represents the risk-free rate.

In 1994, the ratio was revised by Sharpe with substituting R_{RF} by R_B, and the formula became as follows,

$$SR_{R,R_B} = \frac{E(R - R_B)}{\sigma_{R-R_B}}. \tag{3.7}$$

The Sharpe ratio is applied when the assets in the portfolio are risky assets. The Sharpe ratio represents that how many excess return an investor get for each additional risk; if it is positive, it means that the portfolio's rate of return is higher than the risk free rate; if it is negative, it means that the risk free rate is greater than the rate of return. In this way, for each portfolio, we can calculate the Sharpe ratio. The higher the ratio, the better the portfolio.

Although the Sharpe ratio is very simple in calculation, it is still needed to pay attention in application because:

1) It adjusts return with standard deviation. The implicit assumption is that the portfolio examined is the entire investment of the investor. Therefore, the Sharpe ratio can be used as an important basis only when purchasing a fund among a large number

of funds,

2) The Sharpe ratio has no benchmark, thus the size of the value of itself has no meaning , an it's only valuable in comparison with results from other funds.

3.2.3 Jensen's Alpha

The last ratio that we are going to introduce is the Jensen's alpha. It is a measure of the size of the portfolio's excess returns. Similar with the Sharpe ratio, this measure consider both the portfolio's returns and risk as factors. Generally, when we want to evaluate the performance of portfolios, the return is the most direct indicator but it's not comprehensive and relatively simple.

In 1968, the American economist Michael C. Jensen proposed this performance measurement index which is based on the Capital Asset Pricing Model (CAPM). Through comparing the difference between the rate of return of the portfolio in the observed period and the expected rate of return derived from the CAPM, the index can assess the level of performance of the portfolio over the benchmark. And it can be represented in equation as: Actual return of the portfolio = Jensen index (excess return) + return from exposure to market risk. The mathematical formula of Jensen's alpha is as follows,

$$R_{P,t} - R_{f,t} = \alpha_P + \beta_P (R_{M,t} - R_{f,t}) + \varepsilon_{P,t}, \tag{3.8}$$

where α_P is the Jensen's alpha, β_P is the portfolio beta, and $\varepsilon_{P,t}$ represents the error term.

Therefore, the index represents the excess returns from the portfolio's performance that exceeds the market benchmark portfolio. That is, when the Jensen index is higher than 0, it means that the performance of the portfolio is better than the market benchmark portfolio. And the higher the excess part, the better the performance. For fund managers, it shows that the manager has an extraordinary ability in picking stock and has good portfolio performance. On the contrary, if the index is lower than 0, it means that the performance of the portfolio is not good. It can also be explained as the managers have poor ability in picking stock. When the index is equal to 0, it means that the manager's ability in picking stock is on average.

Therefore, investors can use the Jensen's alpha to assess the alternative portfolios. As long as the index is positive at one specific period, even if the portfolio's return is negative, we can still think that the portfolio is a good. On the contrary, the investor should choose other better portfolio.

4. Application of Portfolio Optimization

In the previous chapter, we introduce the approach to estimate returns of individual assets and portfolios, the strategies of portfolio optimization, the theory of backtesting and measures of portfolio performance. In this chapter, we are going to apply these methodologies to actual data.

Firstly, we will describe the data we choose. There are 30 stocks from Hang Seng Index. These data are adjusted close prices of selected stocks, and they are divided into two periods: in-sample and out-of-sample. We start our investment in out-of-sample period, and the initial wealth is 1 HKD. Through maximization of utility function under different value of k (risk attitude of investor), we find out the different compositions of weights of each stock. Then we apply selected strategies to estimate the means of returns and final wealth of out-of-sample portfolios. Finally, we apply different performance measures to compare the performance of strategies, and select the best strategy.

4.1 Data Description

In this section, we are going to describe the data we select. Since the topic of the thesis is optimization of stock portfolios, the data we need are adjusted close prices of different stocks. We choose 30 stocks in Hang Seng Index (HSI) based on the market capitalization and data availability.

Hang Seng Index is an important indicator which reflects the Hong Kong stock market. The Hang Seng Index was officially issued on November 24, 1969. The index is calculated from the market capitalization of 50 HSI constituents and the constituents are blue chip stocks in Hong Kong. The Hang Seng Index is calculated by Hang Seng Index Co., Ltd. and reviewed quarterly to announce the adjustment of constituents. It means that the number of constituents are fixed, which is 50, but the real constituents are changing.

When selecting the stocks, we can go to the official website of Hang Seng

Indexes. We can find the newest complete list of constituents there. Based on the market capitalization, we can get a list of constituents with new order. Because for some of them, the data range we want are not available, thus we skip them, and pick 30 stocks as our data set. The name and abbreviations of each selected stock are shown in table 4.1.

Table 4.1 Name of selected stocks and abbreviations

Name	Abbreviation
Tencent	TC
Industrial and Commercial Bank of China	ICBC
China Construction Bank	CCB
PINGAN	PA
China Mobile	CM
HSBC Holdings	HSBC
Bank of China Limited	BCL
China Life Insurance	CLI
China Petroleum & Chemical	CPC
CNOOC	CNOOC
Bank of Communications	BCC
Sun Hung Kai Properties	SHKP
China Shenhua Energy	CSE
Hang Seng Bank	HSB
BOC Hong Kong	BOC
CITIC	CITIC
Hong Kong Exchanges and Clearing	HKEX
China Overseas Land & Investment	COLI
CK Hutchison Holdings	CKH
MTR Corporation	MTR
China Unicom (Hong Kong)	CU
The Hong Kong and China Gas	HKCG
Country Garden Holdings	CG
Galaxy Entertainment	GE
China Resources Land	CRL
CLP Holdings	CLP
Henderson Land Development	HLD
Link Real Estate Investment Trust	LRE
CK Infrastructure	CKI
Shenzhou International	SZ

source: own elaboration

The adjusted close prices of each stock can be found in the official website of

Yahoo Finance. The data are weekly data between January 4th 2009 and December 30th 2018. Thus there are 522 weeks' prices of each selected stock.

Table 4.2 Mean of returns and standard deviations (weekly)

	Whole period		In-sample		Out-of-sample	
	Mean of return	σ	Mean of return	σ	Mean of return	σ
TC	0.806%	4.399%	1.084%	4.727%	0.529%	4.036%
ICBC	0.258%	4.083%	0.310%	4.626%	0.206%	3.466%
CCB	0.258%	3.785%	0.302%	4.091%	0.213%	3.461%
PAI	0.394%	4.797%	0.383%	5.293%	0.404%	4.256%
CM	0.113%	2.911%	0.139%	3.153%	0.086%	2.655%
HSBC	0.118%	3.601%	0.184%	4.346%	0.052%	2.665%
BCL	0.294%	3.853%	0.428%	4.234%	0.160%	3.434%
CLI	0.062%	4.482%	0.121%	4.642%	0.003%	4.326%
CPC	0.251%	3.931%	0.369%	4.006%	0.134%	3.859%
CNOOC	0.270%	4.537%	0.418%	4.836%	0.123%	4.223%
BCC	0.211%	4.090%	0.184%	4.654%	0.237%	3.447%
SHKP	0.211%	3.492%	0.256%	3.992%	0.167%	2.917%
CSE	0.187%	4.716%	0.283%	4.892%	0.092%	4.542%
HSB	0.236%	2.864%	0.228%	3.323%	0.244%	2.324%
BOC	0.371%	3.603%	0.556%	4.115%	0.187%	3.005%
CITIC	0.181%	4.724%	0.225%	5.802%	0.137%	3.331%
HKEX	0.350%	4.414%	0.359%	4.885%	0.341%	3.897%
COLI	0.351%	5.101%	0.449%	5.624%	0.253%	4.529%
CKH	0.192%	3.355%	0.329%	3.908%	0.055%	2.694%
MTR	0.240%	2.405%	0.260%	2.721%	0.220%	2.048%
CU	0.108%	4.311%	0.231%	4.687%	-0.015%	3.906%
HKCG	0.303%	2.278%	0.385%	2.605%	0.222%	1.900%
CG	0.609%	6.231%	0.626%	6.641%	0.593%	5.808%
GE	0.853%	5.891%	1.708%	6.395%	0.002%	5.217%
CRL	0.419%	5.582%	0.491%	6.224%	0.348%	4.871%
CLP	0.197%	1.801%	0.151%	1.728%	0.243%	1.874%
HLD	0.284%	3.827%	0.323%	4.496%	0.244%	3.027%
LRE	0.453%	2.585%	0.509%	2.650%	0.398%	2.523%
CKI	0.246%	2.433%	0.326%	2.552%	0.166%	2.310%
SZ	0.967%	5.078%	1.398%	5.989%	0.537%	3.934%

source: own elaboration

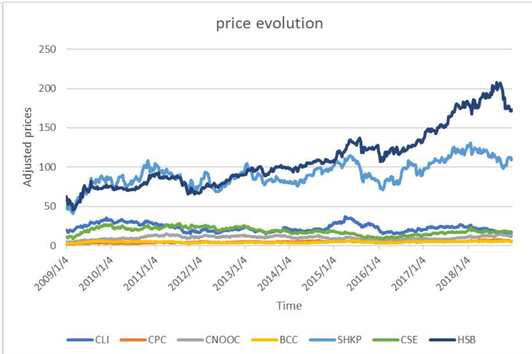
We divide the time range into two halves, half of which is in-sample period (January 4th 2009-December 29th 2013) and the other half is out-of-sample period

(January 5th 2014-December 30th 2018). The initial wealth is 1 HKD. We can estimate mean of returns and standard deviations of each period by applying equations (2.1)、(2.3)、(2.4)、(2.5). The results are shown in the above table 4.2.

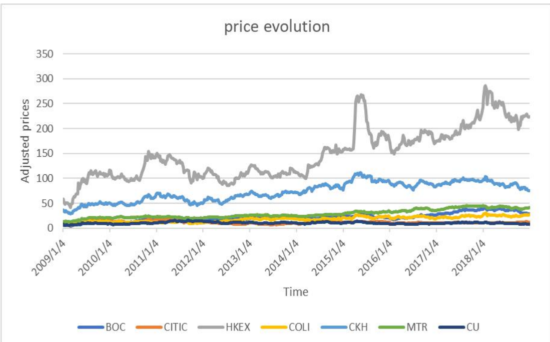
In whole period, the stock with highest mean of return is SZ and the stock with lowest standard deviation is CLP. In in-sample period, the stock with highest mean of return is GE, and the stock with lowest standard deviation is CLP. In out-of-sample period, the stock with highest mean of return and highest standard deviation is CG, and the stock with lowest standard deviation is CLP. Moreover, we make charts of prices evolution of each stock.

Chart 4.1 Evolution of prices

(1)



(2)



(3)

(4)

source: own elaboration

From the first chart, stock prices of TC (Tencent) started to increase in 2014, and increased significantly since 2017, reaching its peak in early 2018 and then falling back. The year 2017 was the most "crazy" year for Tencent's share price - the stock price has risen more than double of previous price, and the market capitalization has been achieved from 2 trillion to 4 trillion in the beginning of 2018. That was where the “Shares king” came from. Behind the growth of stock price, the reason was its profitability that always surprised the market. According to General Accounting Standards (GAAP), in 2017, Tencent's growth rate of net profit of each quarter was above 50%.

In fact, since the first quarter of 2007, Tencent’s “other income” has surged. It has stabilized at around 2%-3% of total revenue before, and has risen to more than 10% in 2018. In the "other income", investment income accounted for the largest proportion. It is believe that the growth of “other income” has masked the growth weakness of Tencent’s main business such as online games, digital content and social advertising. Not only Tencent, but also other stocks in the chart can be found to have different degrees of decline in 2018. Hang Seng Index fell 13.6% for the full year in 2018, and it was the biggest drop since 2011. The Hang Seng Index hit a historic high of 33,484.08 points at the end of January in 2018, and has fallen after it. Tencent fell 22.7% this year, and it was the first annual decline after 2011.

In the last chart, there is an distinctive peak which belongs to GE (Galaxy Entertainment). The principal activities of the company and its subsidiaries (collectively referred to as the “Group”) are the operation of casino for lucky gambling or other forms of gaming in Macao, the provision of hotels and related services and the production, sale and distribution of building materials in Hong Kong, Macao and Mainland China. In 2018, GE has entered its 10th anniversary in Macao, and despite the challenges in the market environment in the second half of the year, it still recorded solid financial results. Revenue for the year increased by 9%. The Group's adjusted EBITDA increased by 5% and profit attributable to shareholders increased by 3% from last year to a record high of HKD 10.3 billion. “Galaxy Macao” was once again the key to the record of the Group's performance. The adjusted EBITDA of the business increased by 12%. The revenue also increased by 18% and the return on investment in 2014 reached 58%. 2015 was the most difficult year for the industry to operate, and the overall market environment was not good. China continued the transform into a consumer-oriented economy and anti-corruption measures continued to be implemented, reducing the consumption in Macao and deterring VIP customers. According to data from the Macao Gaming Inspection and Coordination Bureau, in 2015, Macao gaming revenue fell by 34.3%.

4.2 Naive Strategy

As we have introduced in chapter 2, naive strategy is aimed to make $1/N$ portfolio. It means that every single stock accounts for the same proportion in the portfolio, which is $1/N$. In our sample, we have 30 stocks, so every stock accounts for 1/30. Since we have divided our data set into in-sample and out-of-sample period, our investment begins from January 5th 2014, and the initial wealth is 1 HKD. Then we calculate the portfolio return and wealth evolution under naive strategy.

According to this proportion and formula (2.12), we can get the portfolio returns for each week. According to formula (3.2), we can get the weekly wealth evolution. The results of naive strategy are shown in table 4.3.

Table 4.3 Portfolio returns and wealth evolution in out-of-sample period (weekly)

Date	portfolio return	wealth	Date	portfolio return	wealth
2014/1/5	-0.839%	0.99161	2016/12/11	-3.944%	1.22619
2014/1/12	0.136%	0.99296		…...	
	…...		2017/1/8	2.066%	1.2696
2014/3/2	-1.117%	0.96997	2017/1/15	-0.338%	1.26531
2014/3/9	-4.855%	0.92287		…...	
	…...		2018/1/21	3.348%	1.8923
2015/1/18	2.288%	1.1868	2018/1/28	-1.155%	1.87045
2015/10/11	3.279%	1.18392		…...	
	…...		2018/11/25	2.140%	1.68424
2016/7/24	-0.183%	1.18188		......	
2016/7/31	0.790%	1.19121	2018/12/30	1.311%	1.65228

source: own elaboration

Our initial wealth is 1 HKD, meaning that we use 1 HKD to invest in the portfolio. After 1 week, the wealth become 1*[1+(-0.0839%)]=0.99161. By analogy, we can figure out the rest of wealth. After 5 years' investment, the final wealth is 1.65228 HKD, thus we earn 0.65228 HKD in this investment. Also, during this period, the lowest wealth is 0.92287 on March 9th 2014, and the highest wealth is 1.8923 on January 21st 2018. Also we make a chart to show the wealth evolution intuitively. In chart 4.2, we can find that the wealth is generally increasing. But it has decline during 2015, especially in July. The reason for the decline in wealth is bear market in Hong Kong. The reasons of plunge in Hong Kong stocks came from different aspects. On the one hand, A shares (shares in mainland China) continued to fall, causing the decline of part of Hong Kong stocks (stocks of mainland China enterprises which are listed in Hong Kong stock exchange) and the confidence of mainland investors. On the other hand, the Greek referendum vetoed the rescue plan, which caused many stock market indexes in the world to be frustrated, causing outflow of some capital. The funds were biased towards safe-haven assets such as US Treasury Bond.

Chart 4.2 Wealth evolution of Naive strategy

source: own elaboration

After obtaining results under naive strategy, we calculate performance measures that we select. First, we choose the Hong Kong 10Y Government Bond as risk free asset, and it has a 1.675% annual yield, so the weekly risk free rate is 0.0322%. According to table 4.3, we can estimate the mean of return of portfolio which is 0.219%, and standard deviation which is 2.315% in out-of-sample period.

Based on formula (3.5), we can calculate maximum drawdown in Excel. Through formula (3.6), Sharpe ratio can be calculated as:

$$SR_{R,R_{RF}} = \frac{E(R) - R_{RF}}{\sigma_R} = \frac{0.219\% - 0.0322\%}{2.315\%} = 0.081 \quad . \qquad (4.1)$$

To calculate Jensen's alpha, it's more complicated than previous two measures. The calculation requires the following inputs:

$R_{P,t}$: the realized return on the portfolio,

$R_{M,t}$: the market return or the the realized return of the market index,

$R_{f,t}$: the risk-free rate of return,

β_p : the beta of the portfolio.

First we have to know the value of R_M which is expected return of the HSI between 2009 and 2018. We can download the weekly adjusted close prices on the official website of Yahoo Finance, and calculate the mean of weekly returns. The result calculated by Excel is 0.1493%. Then we have to calculate β of the portfolio. It can be calculated as the weighted average of betas of all stocks. The beta of i-th stock can be calculated as following:

$$\beta_i = \frac{E(R_i) - R_f}{R_M - R_f} . \tag{4.2}$$

And the weights of each stock are the same (1/30), thus we can calculate the β_P in Excel, and the result is 3.3196. Weekly return of portfolio and weekly risk free rate which can be found in previous text are 0.219% and 0.0322%. Now base on formula (3.8), we can calculate the Jensen's alpha as follows:

$$\alpha_P = 0.219\% - [0.0322\% + 3.3196 * (0.1493\% - 0.0322\%)] = -0.202\%, \tag{4.3}$$

the result is negative but is extremely close to 0.

Table 4.4 Result from naive strategy

Final wealth (HKD)	1.652
Weekly portfolio return	0.219%
Annual portfolio return	11.408%
Weekly standard deviation	2.315%
Annual standard deviation	16.693%
Sharpe ratio	0.081
Maximum drawdown	29.545%
Jensen's alpha	-0.202%

source: own elaboration

4.3 Portfolio Optimization of Markowitz Model

The second strategy we are going to apply is Markowitz model. In this chapter, calculations are made in two periods separately. Unlike naive strategy, the weights of stocks are unknown. We use Markowitz model in in-sample period to construct the efficient frontier. In out-of-sample period, we focus on the influences of value of parameter k. We back-test the portfolio optimization for small value of k and large value of k. We apply utility function under different value of k to find out the weights of portfolios, and apply them in out-of-sample period. According to the description of utility function in chapter 2, when $k \to \infty$, the optimal portfolio is approaching the minimum-variance portfolio. So we can back-test the minimum variance portfolio in this part too.

4.3.1 Portfolio Optimization in In-sample Period

In this part, Markowitz model is applied in in-sample period. According to description in chapter 2, Markowitz proposed that expected return, and variance of return of the portfolio should be the criteria for portfolio selection. On this basis, in order to find out the efficient set of Markowitz model, we need to get the value of expected return of portfolios under specified weights and standard deviations.

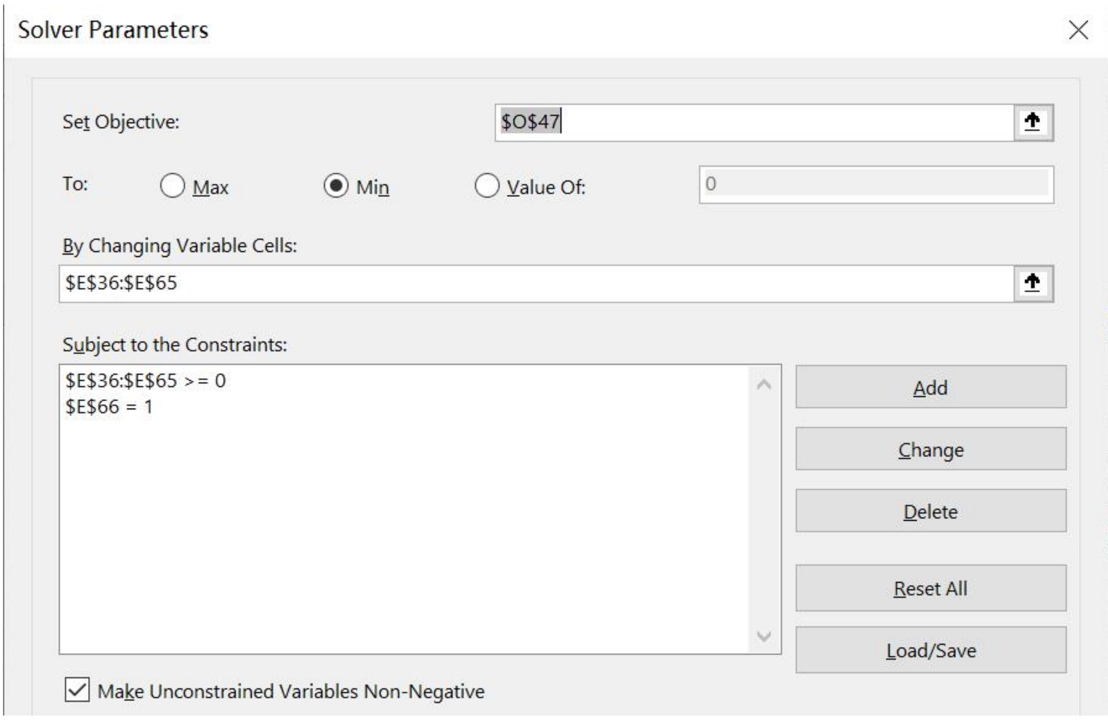
The original data we use are the adjusted close stock prices in in-sample period (January 4th 2009-December 29th 2013), so there are 261 weekly prices and 260 weekly returns of each stock. Based on these data, we can estimate 30 expected returns because we have 30 stocks and 30 standard deviations.

Table 4.5 shows the results of calculation under Markowitz model with in-sample period. Because of the limit of space, we choose the most important part to show in the table. When we calculate the expected return of the particular efficient portfolio ($E(R_P)_{cal}$ in the table) and variance (σ^2), the referenced cells (*B3:AE32*) is sample covariance matrix which is calculated from population covariance. The population covariance matrix is calculated based on weekly stock returns. And the optimal composition of each portfolio is found by the *Solver* module in Excel.

Points on the efficient set are all alternative choices of efficient portfolios, without risk attitude of investors, we can only draw the efficient set. But the start point and end point of the line are fixed, which is minimum variance portfolio *A* and maximum return portfolio *B*. We choose to show portfolios *C* to *H* as alternative portfolios on the efficient set. In fact, we can generate many portfolios based on Markowitz model.

According to objective functions and constraints of Markowitz model in section 2.3.1, we use S*olver* to solve the problems. To solve the minimization of standard deviation problem, we set constraints in *Solver* as in chart 4.3.

Chart 4.3 S*olver* parameters-Minimum risk portfolio



source: own elaboration

When solving the problem of maximum return portfolio *B*, the procedure is similar, but we have to change the "*min*" to "*max*".

Table 4.5 Markowitz model (in-sample period)

	A	B	C	D	E	F	G	H	I	J	K	L
34		In-sample			Vector of variable x							
35		E(Ri)	σ		A	C	D	E	F	G	H	B
36	TC	1.084%	4.727%	TC	0.00%	3.61%	0.45%	10.02%	12.66%	14.72%	13.71%	0.00%
37	ICBC	0.310%	4.626%	ICBC	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
38	CCB	0.302%	4.091%	CCB	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
39	PAI	0.383%	5.293%	PA	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
40	CM	0.139%	3.153%	CM	4.21%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
41	HSBC	0.184%	4.346%	HSBC	0.11%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
42	BCL	0.428%	4.234%	BCL	0.00%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
43	CLI	0.121%	4.642%	CLI	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
44	CPC	0.369%	4.006%	CPC	1.78%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
45	CNOOC	0.418%	4.836%	CNOOC	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
46	BCC	0.184%	4.654%	BCC	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
47	SHKP	0.256%	3.992%	SHKP	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
48	CSE	0.283%	4.892%	CSE	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
49	HSB	0.228%	3.323%	HSB	2.03%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
50	BOC	0.556%	4.115%	BOC	0.00%	0.40%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
51	CITIC	0.225%	5.802%	CITIC	1.66%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
52	HKEX	0.359%	4.885%	HKEX	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
53	COLI	0.449%	5.624%	COLI	0.34%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
54	CKH	0.329%	3.908%	CKH	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
55	MTR	0.260%	2.721%	MTR	0.93%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
56	CU	0.231%	4.687%	CU	0.00%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
57	HKCG	0.385%	2.605%	HKCG	1.29%	1.78%	2.48%	0.00%	0.00%	0.00%	0.00%	0.00%
58	CG	0.626%	6.641%	CG	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
59	GE	1.708%	6.395%	GE	0.00%	6.28%	15.04%	19.23%	28.07%	37.15%	48.44%	100.00%
60	CRL	0.491%	6.224%	CRL	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
61	CLP	0.151%	1.728%	CLP	49.49%	38.84%	19.42%	1.16%	0.00%	0.00%	0.00%	0.00%
62	HLD	0.323%	4.496%	HLD	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
63	LRE	0.509%	2.650%	LRE	16.97%	21.89%	27.55%	30.90%	26.33%	18.56%	0.00%	0.00%
64	CKI	0.326%	2.552%	CKI	18.76%	20.17%	22.26%	22.51%	10.49%	0.00%	0.00%	0.00%
65	SZ	1.398%	5.989%	SZ	2.42%	6.99%	12.80%	16.17%	22.45%	29.57%	37.86%	0.00%
66				Sum	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%

	M	N	O	P	Q	R	S	T	U	V
35				=O39		=(V39-O39)/7		=SUMPRODUCT($B$36:$B$65,L36:L65)		
37		Equidistant interval		0.00203		=O38+$P$37				
38		$E(R_p)$gen	0.286%	0.489%	0.693%	0.896%	1.099%	1.302%	1.505%	1.708%
39		$E(R_p)$calc	0.286%	0.489%	0.693%	0.896%	1.099%	1.302%	1.505%	1.708%
40		σ_p^2	0.020%	0.023%	0.037%	0.058%	0.093%	0.143%	0.212%	0.409%
41		σ_p	1.397%	1.522%	1.911%	2.417%	3.054%	3.788%	4.606%	6.395%
42				=SQRT(O40)						
43	**Constraints**	C1				$\sum_i x_i$=1				
44		C2				$x_i \geq 0$			=SUMPRODUCT(L36:L65, MMULT($B$3:$AE$32,L36 :L65))	
45		C3				E(Rp)gen=E(Rp)calc				
47	**Objective function**		1.397%	1.522%	1.911%	2.417%	3.054%	3.788%	4.606%	1.708%
48						↓				↓
49			=C73			minimizing standard deviation			=V39	maximize
50										the
66	=SUM(M36:M65)									

source: own elaboration

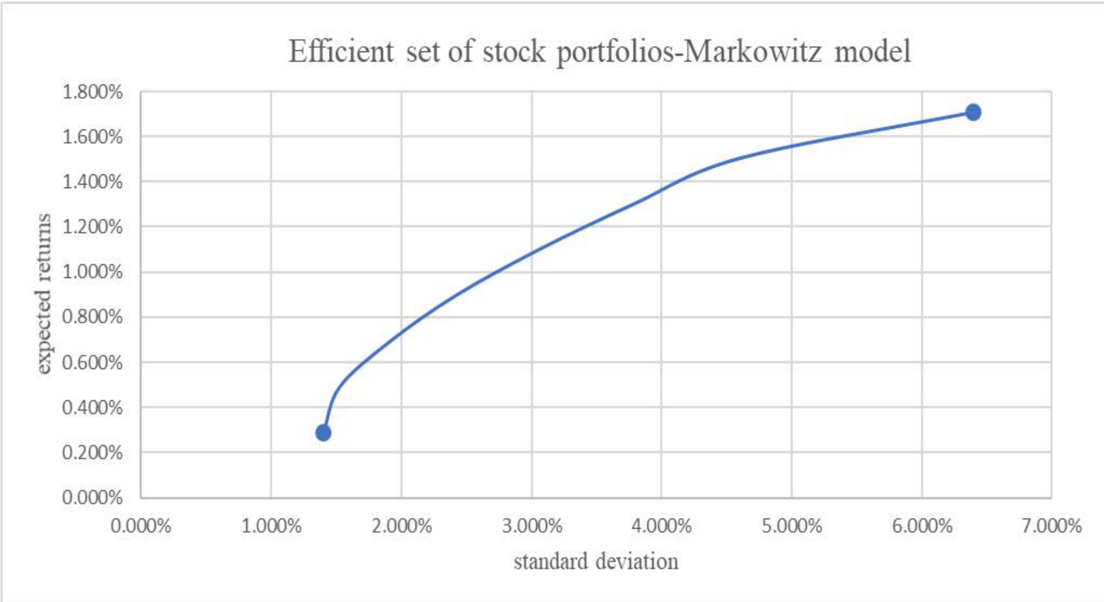
After calculating values of standard deviation and expected return of each portfolio, we can make a chart of efficient frontier.

Table 4.6 Standard deviation and expected return (in-sample period)

Portfolio	A	C	D	E	F	G	H	B
σ	1.397%	1.522%	1.911%	2.417%	3.054%	3.788%	4.606%	6.395%
$E(R_P)$	0.286%	0.489%	0.693%	0.896%	1.099%	1.302%	1.505%	1.708%

source: own elaboration

Chart 4.4 Efficient set under Markowitz model (in-sample period)



source: own elaboration

As we can see, the efficient frontier starts with the minimum variance portfolio A which has the lowest standard deviation (1.397%) and lowest expected return (0.286%).

4.3.2 Backtesting of Markowitz Model with Small Value of *k*

As we have introduced in sub-chapter 2.7, we can confirm the optimal portfolio with the knowledge of particular person's risk aversion level. Thus we use expected utility criterion to determine the optimal portfolio with different value of parameter k. In this section, we apply back-test to Markowitz model with small value of k. The investment period is out-of-sample period (January 5th 2014-December 30th 2018).

By setting different values of k, we can get weights of each stock from in-sample period. The small values of k that we select are 0, 2, 4, 6, 8.

Similar to calculation in table 4.5 of Markowitz model, we also use S*olver* to find optimal portfolio with respect to the utility function. According to constraints in sub-chapter 2.7, we get objective functions and constraints. Similar to *Solver* in Markowitz model and procedure in 4.5, we only need to change the objective functions and constraints of utility function according to equation (2.25). Table 4.7 is the procedure of how to determine the weights with respect to utility criterion under different value of k.

Table 4.7 Utility function with different small value of k

	B	C	D	E	F	G	H	I
34	In-sample		k	0	2	4	6	8
35	E(Ri)	σ	TC	0.00%	2.93%	14.03%	11.24%	9.42%
36	1.084%	4.727%	ICBC	0.00%	0.00%	0.00%	0.00%	0.00%
37	0.310%	4.626%	CCB	0.00%	0.00%	0.00%	0.00%	0.00%
38	0.302%	4.091%	PA	0.00%	0.00%	0.00%	0.00%	0.00%
39	0.383%	5.293%	CM	0.00%	0.00%	0.00%	0.00%	0.00%
40	0.139%	3.153%	HSBC	0.00%	0.00%	0.00%	0.00%	0.00%
41	0.184%	4.346%	BCL	0.00%	0.00%	0.00%	0.00%	0.00%
42	0.428%	4.234%	CLI	0.00%	0.00%	0.00%	0.00%	0.00%
43	0.121%	4.642%	CPC	0.00%	0.00%	0.00%	0.00%	0.00%
44	0.369%	4.006%	CNOOC	0.00%	0.00%	0.00%	0.00%	0.00%
45	0.418%	4.836%	BCC	0.00%	0.00%	0.00%	0.00%	0.00%
46	0.184%	4.654%	SHKP	0.00%	0.00%	0.00%	0.00%	0.00%
47	0.256%	3.992%	CSE	0.00%	0.00%	0.00%	0.00%	0.00%
48	0.283%	4.892%	HSB	0.00%	0.00%	0.00%	0.00%	0.00%
49	0.228%	3.323%	BOC	0.00%	0.00%	0.00%	0.00%	0.00%
50	0.556%	4.115%	CITIC	0.00%	0.00%	0.00%	0.00%	0.00%
51	0.225%	5.802%	HKEX	0.00%	0.00%	0.00%	0.00%	0.00%
52	0.359%	4.885%	COLI	0.00%	0.00%	0.00%	0.00%	0.00%
53	0.449%	5.624%	CKH	0.00%	0.00%	0.00%	0.00%	0.00%
54	0.329%	3.908%	MTR	0.00%	0.00%	0.00%	0.00%	0.00%
55	0.260%	2.721%	CU	0.00%	0.00%	0.00%	0.00%	0.00%
56	0.231%	4.687%	HKCG	0.00%	0.00%	0.00%	0.00%	0.00%
57	0.385%	2.605%	CG	0.00%	0.00%	0.00%	0.00%	0.00%
58	0.626%	6.641%	GE	100.00%	57.33%	32.79%	23.17%	18.10%
59	1.708%	6.395%	CRL	0.00%	0.00%	0.00%	0.00%	0.00%
60	0.491%	6.224%	CLP	0.00%	0.00%	0.00%	0.00%	5.65%
61	0.151%	1.728%	HLD	0.00%	0.00%	0.00%	0.00%	0.00%
62	0.323%	4.496%	LRE	0.00%	0.00%	23.57%	29.18%	29.82%
63	0.509%	2.650%	CKI	0.00%	0.00%	3.45%	17.80%	22.17%
64	0.326%	2.552%	SZ	0.00%	39.73%	26.16%	18.61%	14.85%
65	1.398%	5.989%	sum	100.00%	100.00%	100.00%	100.00%	100.00%

	J	K	L	M	N	O
34						
35	$E(R_p)$	1.708%	1.567%	1.209%	0.984%	0.852%
36	σ^2	0.409%	0.240%	0.119%	0.072%	0.053%
37	Constraints	$\sum_{i=1}^{N} x_i=1$				
38		$x_i \geq 0, i = 1, \dots, N$				
39	Objective function	0.017083	0.010873	0.007349	0.005539	0.004303
40						=O35-I34*O36

source: own elaboration

Then we need to apply weights to out-of-sample period. According to formula (3.1) and (3.2), we can get weekly portfolio returns and accumulated wealth in out-of-sample period with different value of k.

According to table 4.8, we have lowest final wealth 0.705 when k=0, this is the only situation that we have a loss after investment in the out-of-sample period. It's worth noting that when k=0, the optimal portfolio is the maximum return portfolio. This point represents the highest expected return and highest standard deviation (risk). We estimate the expected return of selected stocks in the in-sample-period, and according to maximum return portfolio strategy, we invest all the money into the stock with highest return. We believe the result in the in-sample period will continue in the future, but in fact, the mean of return in the out-of-sample is quite different from the in-sample period and it causes bias. In the in-sample period, the stock with highest expected return is GE (1.708%), but in the out-of-sample period, the mean of return of GE falls to 0.002%. That's the reason why we have lowest final wealth when *k*=0 in out-of-sample period. The highest final wealth is under the condition when *k*=2. When *k*=4, 6, 8, we have similar value of final wealth.

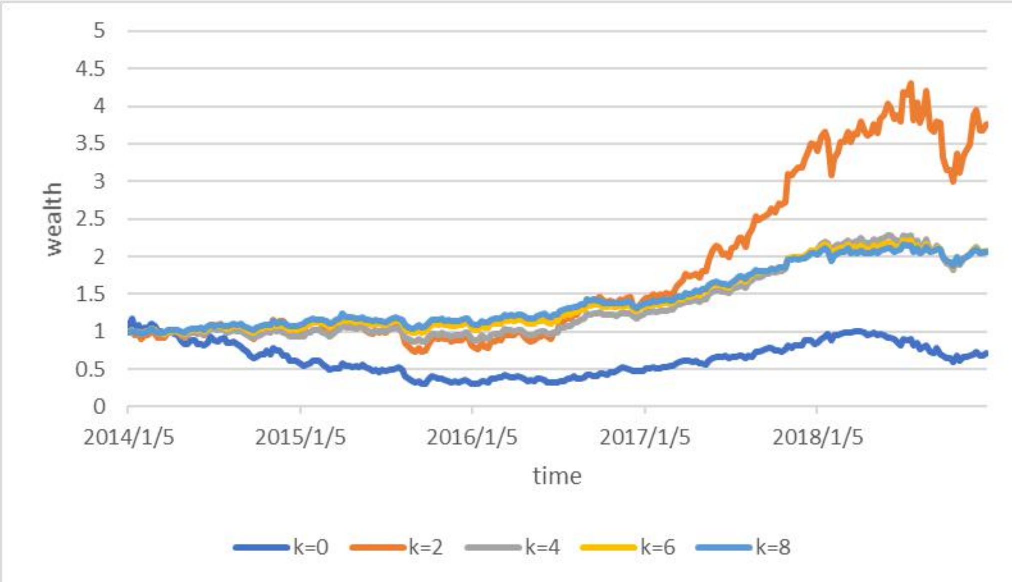
Table 4.8 Portfolio returns and wealth with small value of *k*

	k=0		*k*=2		*k*=4		*k*=6		*k*=8	
Date	returns	wealth	returns	wealth	returns	wealth	returns	wealth	returns	wealth
14/1/5	6.23%	1.062	-2.00%	0.980	-0.17%	0.998	-1.33%	0.987	-1.70%	0.983
14/1/12	10.86%	1.178	4.22%	1.021	2.85%	1.027	2.22%	1.009	1.91%	1.002
…...										
14/12/28	-1.83%	0.607	-0.49%	1.011	-0.56%	0.934	-0.48%	1.027	-0.44%	1.073
15/1/4	-6.30%	0.568	-2.88%	0.982	0.74%	0.941	1.13%	1.039	1.17%	1.085
…...										
15/12/27	-1.41%	0.346	-0.04%	0.933	0.08%	0.981	0.38%	1.116	0.53%	1.177
16/1/3	-9.82%	0.312	-13.09%	0.811	-7.35%	0.909	-6.10%	1.048	-5.44%	1.113
…...										
16/12/25	2.11%	0.478	5.07%	1.349	2.95%	1.214	2.80%	1.305	2.55%	1.344
17/1/1	0.59%	0.481	4.92%	1.416	1.41%	1.231	1.67%	1.327	1.85%	1.369
…...										
17/12/31	-5.42%	0.839	-0.72%	3.482	-0.28%	2.074	0.25%	2.077	0.43%	2.045
18/1/7	1.43%	0.851	-2.05%	3.411	0.79%	2.091	-0.32%	2.071	-0.79%	2.028
…...										
18/12/23	0.52%	0.687	0.00%	3.685	-0.83%	2.038	-0.10%	2.056	0.19%	2.051
18/12/30	2.57%	0.705	2.07%	3.761	1.17%	2.062	0.87%	2.073	0.67%	2.064

source: own elaboration

From chart 4.5, we can find that the trends of wealth of every strategy is increasing in 2017, and become decrease in 2018. The reason is the performance of the whole market. The launch of the Hong Kong stock bull market 2017 was mainly due to the improvement of fundamentals. When the Chinese economy entered the "new normal" and structural upgrading was accelerated, the best batch of Chinese companies would emerge as the global industry leader. Among them, the profits of the new economic companies represented by Tencent had risen more than expected, and the stock price increased significantly. But in 2018, Hang Seng Index fell 13.6% for the full year, and it was the biggest drop since 2011.

Chart 4.5 Wealth evolution with different small values of *k*



source: own elaboration

Then we need to calculate the performance measures of strategies with different small values of *k*, and the calculation procedure is same as the procedure in naive strategy in chapter 4.2. The only difference is that we get different weights of stocks, we we need to change the weights when calculate β_P in Jensen's alpha. The results can be found in table 4.9.

Table 4.9 Results from different small value of *k*

	k=0	*k*=2	*k*=4	*k*=6	*k*=8
Final wealth (HKD)	0.705	3.761	2.062	2.073	2.064
Weekly portfolio return	0.002%	0.628%	0.315%	0.305%	0.299%
Annual portfolio return	0.093%	32.636%	16.370%	15.885%	15.555%
Weekly standard deviation	5.217%	4.882%	2.734%	2.273%	2.055%
Annual standard deviation	37.622%	35.205%	19.712%	16.392%	14.821%
Sharpe ratio	-0.006	0.122	0.103	0.120	0.130
Maximum drawdown	74.579%	39.366%	22.178%	17.767%	15.825%
Jensen's alpha	-1.707%	-0.939%	-0.894%	-0.679%	-0.552%

source: own elaboration

From above table, we can find that when k=0, we have the lowest values of Sharpe Ratio and Jenen's alpha, highest value of Maximum drawdown. When k=8, we have the highest values of Sharpe ratio and Jensen's alpha, lowest value of Maximum drawdown. Thus the strategy of k=8 is the best among these 5 strategies.

4.3.3 Backtesting of Markowitz Model with Large Value of *k*

According to the description of utility function in chapter 2, when $k\to\infty$, the optimal portfolio is approaching the minimum-variance portfolio. So this section is the backtesting of the minimum variance portfolio too. In this section, we apply backtesting to Markowitz model with large value of k. The investment period is out-of-sample period (January 5th 2014-December 30th 2018). The procedure is similar to 4.3.1. Firstly, under large value of k, we use utility function to find the weights of stocks from in-sample period. Next, we apply weights to out-of-sample period to get the weekly portfolio return and accumulated wealth. The weights of stocks under each strategy can be found in annex. Table 4.10 shows the results of the procedure.

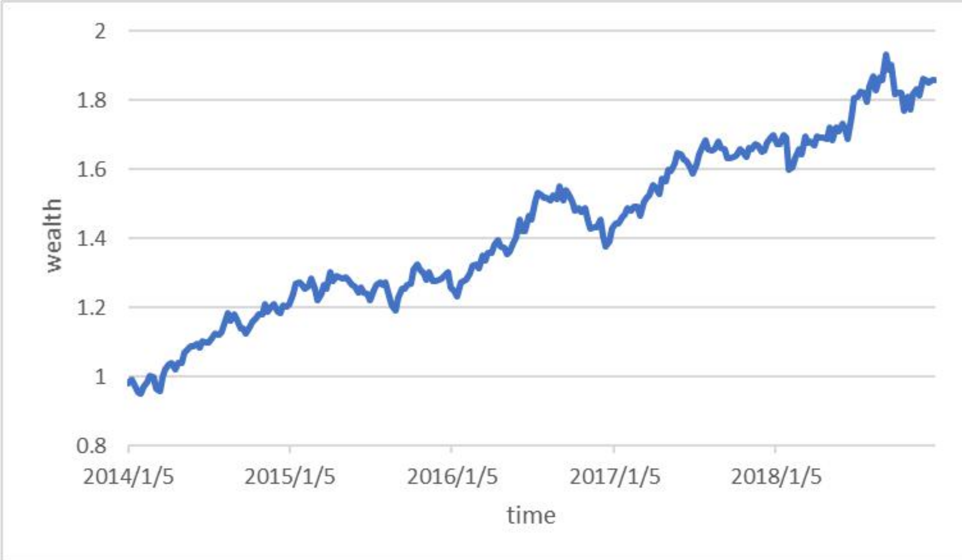
Table 4.10 Portfolio returns and wealth with large value of k

	k=99999			k=99999	
Date	portfolio return	wealth	Date	portfolio return	wealth
2014/1/5	-1.86%	0.981			
2014/1/12	0.78%	0.989	2016/12/25	0.84%	1.390
…...			2017/1/1	2.68%	1.427
2014/12/28	-0.13%	1.204	…...		
2015/1/4	0.45%	1.209	2017/12/31	0.52%	1.699
…...			2018/1/7	-1.65%	1.671
2015/12/27	0.78%	1.303	…...		
2016/1/3	-3.49%	1.257	2018/12/23	0.38%	1.858
…...			2018/12/30	-0.01%	1.858

source: own elaboration

As we have mentioned, when $k\to\infty$, the optimal portfolio is approaching the minimum-variance portfolio, so we should have almost lowest portfolio return. The wealth with large value of k is between 0.952 HKD and 1.933 HKD. Also we make a chart to show the wealth evolution. The general trend is increasing slowly and steadily.

Chart 4.6 Wealth evolution with large value of k



source: own elaboration

Based on table 4.10, we can get the value of weekly portfolio return and weekly standard deviation. Similar to calculation in 4.3.2, we can get the result of performance measures.

Table 4.11 Results from large value of k

	k=99999
Final wealth (HKD)	1.858
Weekly portfolio return	0.250%
Annual portfolio return	13.018%
Weekly standard deviation	1.603%
Annual standard deviation	11.562%
Sharpe ratio	0.136
Maximum drawdown	11.180%
Jensen's alpha	-0.037%

source: own elaboration

Compared to table 4.9, under the condition that k=99999, the values of Sharpe ratio and Jensen’s alpha are the highest and the value of Maximum drawdown is the lowest. Also the final wealth is 1.858 which is almost double of initial wealth. So the strategy with large value of k under Markowitz model is considered as a good choice.

4.4 Portfolio Optimization of Tobin Model

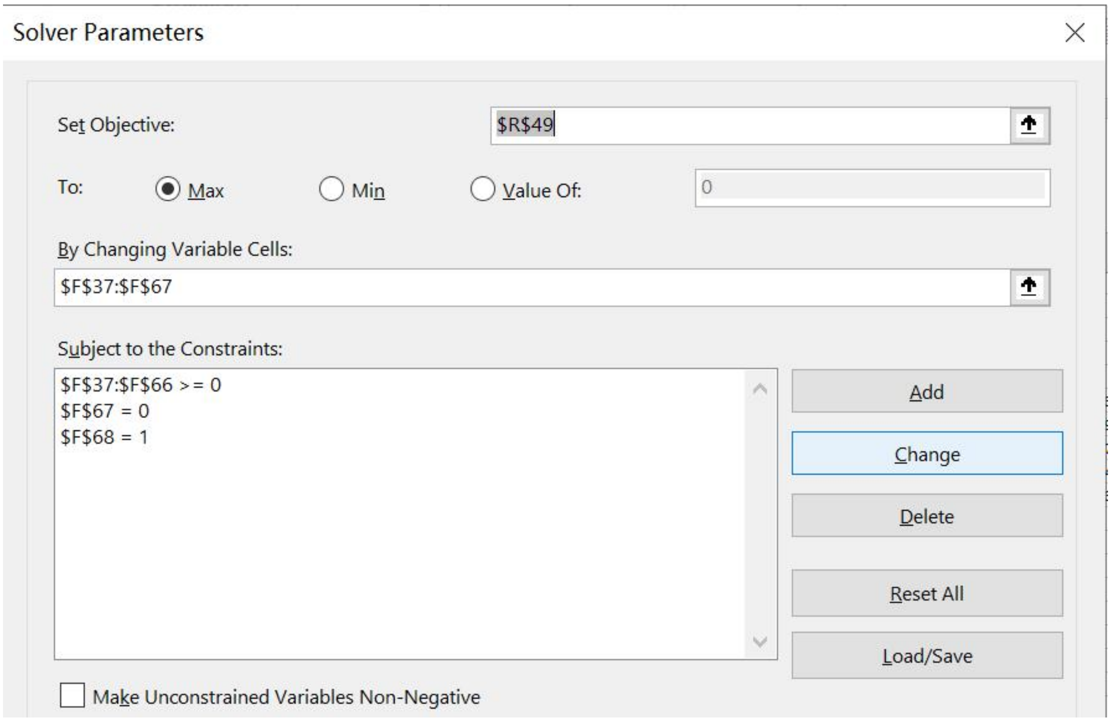
According to chapter 2.5, there are four possibilities for portfolios with different conditions. In the case that short selling is not allowed while risk-free investment is not allowed either, we can apply Markowitz model, and we already applied it in chapter 4.3. In the case that short selling is allowed but risk-free investment is not allowed, we can apply Black's model. But this model is quite similar to Markowitz model, even if we need to change the value range of weights, the matrix of expected return and standard deviation and covariance matrix will be the same. So we apply another case, short selling is not allowed but risk free investment is allowed. In this case, the representative is Tobin model. In this model, we add risk-free asset to the portfolios. Similar to the procedure in chapter 4.3, we firstly apply Tobin model into in-sample period to construct new efficient set. In backtesting, we apply utility function to find weights from in-sample period under different value of *k*, then apply weights to out-of-sample period.

4.4.1 Portfolio Optimization in In-sample Period

As we have mentioned before, we choose Hong Kong 10-year government bond as the risk-free asset. The weekly risk-free rate is 0.0322%, the standard deviation is 0. The covariance between risk free rate asset and other assets is 0. Then we can get new matrix of standard deviation and expected return, and new covariance matrix. According to constraints in chapter 2.5, we solve problems of market portfolio and efficient portfolio using *Solver* module. The solver parameters are shown in chart 4.7 and 4.8.

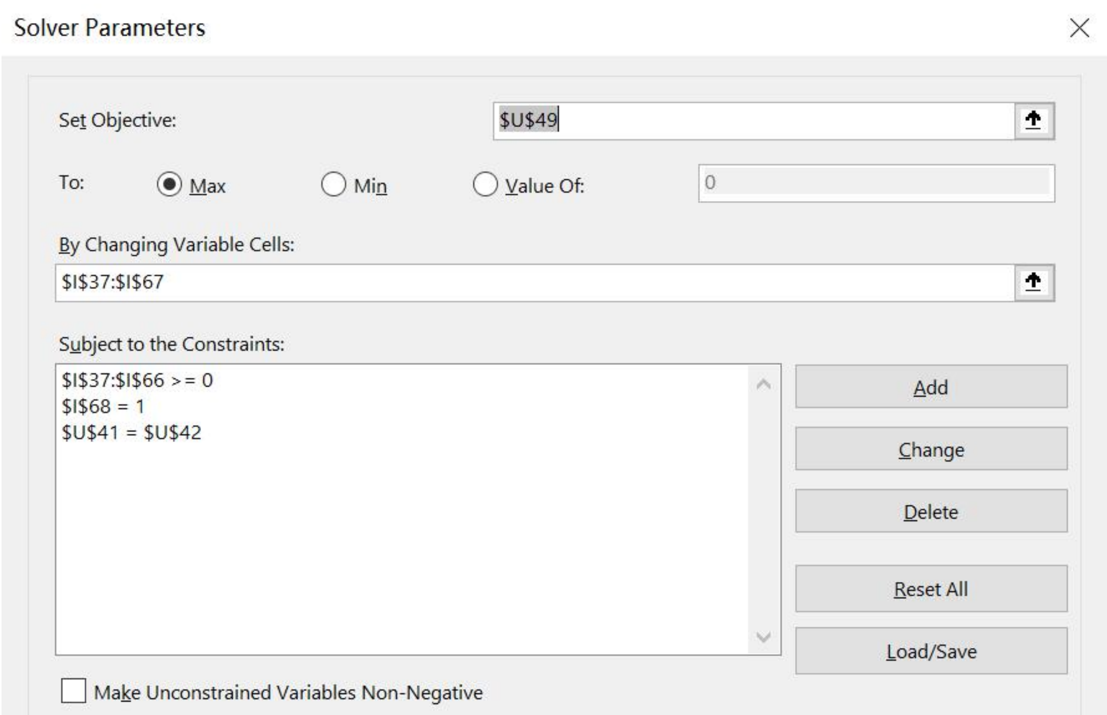
Table 4.12 shows the calculation of Tobin model in Excel. In the table, *M* represents for the market portfolio, *F* presents portfolio that only invest into risk-free asset, portfolio *A* to *H* are efficient portfolios. Referenced cells (*B3:AF33*) represent the covariance matrix. Because of limitation in space, we choose to show efficient portfolios *A* to *H* although we can generate more portfolios and we don't show the covariance matrix.

Chart 4.7 *Solver* parameters-Market portfolio



source: own elaboration

Chart 4.8 *Solver* parameters-Efficient portfolio



source: own elaboration

Table 4.12 Tobin model (in-sample period)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
35		In-sample			vector of variables x										
36		E(Ri)	σ			M	F	A	B	C	M	D	E	G	H
37	TC	1.084%	4.727%		TC	10.03%	0.00%	2.51%	5.01%	7.53%	10.03%	12.54%	15.04%	17.53%	20.05%
38	ICBC	0.310%	4.626%		ICBC	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
39	CCB	0.302%	4.091%		CCB	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
40	PAI	0.383%	5.293%		PAI	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
41	CM	0.139%	3.153%		CM	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
42	HSBC	0.184%	4.346%		HSBC	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
43	BCL	0.428%	4.234%		BCL	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
44	CLI	0.121%	4.642%		CLI	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
45	CPC	0.369%	4.006%		CPC	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
46	CNOOC	0.418%	4.836%		CNOOC	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
47	BCC	0.184%	4.654%		BCC	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
48	SHKP	0.256%	3.992%		SHKP	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
49	CSE	0.283%	4.892%		CSE	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
50	HSB	0.228%	3.323%		HSB	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
51	BOC	0.556%	4.115%		BOC	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
52	CITIC	0.225%	5.802%		CITIC	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
53	HKEX	0.359%	4.885%		HKEX	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
54	COLI	0.449%	5.624%		COLI	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
55	CKH	0.329%	3.908%		CKH	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
56	MTR	0.260%	2.721%		MTR	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
57	CU	0.231%	4.687%		CU	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
58	HKCG	0.385%	2.605%		HKCG	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
59	CG	0.626%	6.641%		CG	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
60	GE	1.708%	6.395%		GE	19.39%	0.00%	4.85%	9.70%	14.55%	19.39%	24.24%	29.09%	33.95%	38.79%
61	CRL	0.491%	6.224%		CRL	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
62	CLP	0.151%	1.728%		CLP	1.80%	0.00%	0.46%	0.81%	1.34%	1.80%	2.26%	2.70%	3.16%	3.60%
63	HLD	0.323%	4.496%		HLD	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
64	LRE	0.509%	2.650%		LRE	30.65%	0.00%	7.65%	15.32%	23.02%	30.65%	38.32%	45.90%	53.68%	61.27%
65	CKI	0.326%	2.552%		CKI	22.42%	0.00%	5.59%	11.22%	16.77%	22.42%	27.99%	33.59%	39.19%	44.88%
66	SZ	1.398%	5.989%		SZ	15.71%	0.00%	3.93%	7.86%	11.78%	15.71%	19.63%	23.59%	27.48%	31.42%
67	R_F	0.0322%	0.000%		R_F	0.00%	100.00%	75.01%	50.08%	25.03%	0.00%	-25.00%	-49.93%	-74.99%	-100.01%
68					sum	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%

	P	Q	R	S	T	U	V	W	X	Y	Z	AA
38		equidistant interval		0.006012								
39		E(Rp)calc	0.8915%	0.0322%	0.2470%	0.4618%	0.6767%	0.8915%	1.1063%	1.3211%	1.5359%	1.7508%
40		σ_p^2	0.0578%	0.0000%	0.0036%	0.0145%	0.0325%	0.0578%	0.0904%	0.1301%	0.1771%	0.2314%
41		σ_p	2.4050%	0.0000%	0.6012%	1.2025%	1.8037%	2.4050%	3.0062%	3.6075%	4.2087%	4.8100%
42		σ_{P-gen}	2.4050%	0.0000%	0.6012%	1.2025%	1.8037%	2.4050%	3.0062%	3.6075%	4.2087%	4.8100%
43		E(Rp)CML	0.8915%	0.0322%	0.2470%	0.4618%	0.6767%	0.8915%	1.1063%	1.3211%	1.5359%	1.7508%
45	**Constraints**	C1					$x_F+\sum_k^N x_k=1$					
46		C2					$x_k \geq 0, for\ k = 1,2,\dots,N$					
47		C3	$x_F=0$			$-\infty \leq x_F \leq \infty$				$-\infty \leq x_F \leq \infty$		
48		C4				$\sigma_p=\sigma_{P-gen}$				$\sigma_p=\sigma_{P-gen}$		
49	Objective function		35.73%		0.2%	0.4618%	0.6767%		1.1063%	1.3211%	1.5359%	1.7508%
50			↓			↓				↓		
51			maximizing the slope					maximizing the expected return				

=R41*2/8

=SUMPRODUCT(H37:H67,MMULT($B$3:$AF$33,H37:H67))

=SUMPRODUCT($B$37:$B$67,O37:O67)

=R39

=S38

=T42+$S$38

=(R39-B67)/R41

=T39

source: own elaboration

The expected returns and standard deviations are shown in table 4.13. Portfolio *F* only consists of risk-free-asset, so the expected return equals to risk-free rate and standard deviation (the risk) equals to 0. Based on results in table 4.13, we can draw the efficient frontier in chart 4.9.

Table 4.13 Standard deviation and expected return (in-sample period)

	F	A	B	C	M	D	E	G	H
σ	0.000%	0.601%	1.202%	1.804%	2.405%	3.006%	3.607%	4.209%	4.810%
$E(R_P)$	0.032%	0.247%	0.462%	0.677%	0.891%	1.106%	1.321%	1.536%	1.751%

source: own elaboration

Chart 4.9 Efficient set (in-sample period)

source: own elaboration

The efficient set of Tobin model is an upward line. The set of efficient portfolios starts with the risk-free portfolio which has the lowest standard deviation (0%) and lowest expected return (0.032%).

4.4.2 Backtesting of Tobin Model with Small Value of *k*

After analysis in the in-sample period, we apply backtesting in this and next section. In this section, we use strategies of different small value of *k*. After applying Tobin model, we get new matrix of expected return and standard deviation, and new matrix of covariance. On this basis, by applying utility function, with certain value of *k*, we can get the optimal portfolios. When we apply the utility function, we add a new constraint: $x_k \geq -1$. The other procedures are same as in backtesting of Makowitz model. We get table 4.14 which shows the portfolio return and wealth under different small value of *k*.

Table 4.14 Portfolio returns and wealth under small value of *k*

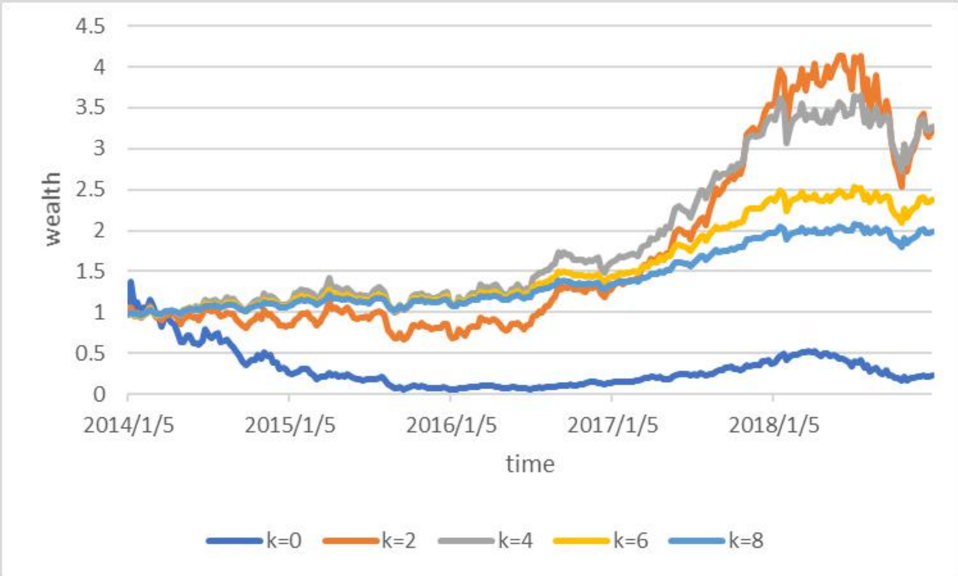
	k=0		*k*=2		*k*=4		*k*=6		*k*=8	
Date (Y/M/D)	returns	wealth	returns	wealth	returns	wealth	returns	wealth	returns	wealth
14/1/5	12.42%	1.124	-0.37%	0.996	-3.20%	0.968	-2.12%	0.979	-1.58%	0.984
14/1/12	21.69%	1.368	5.66%	1.053	3.66%	1.003	2.45%	1.003	1.85%	1.002
…...										
14/12/28	-3.70%	0.305	-1.15%	0.830	-0.86%	1.089	-0.56%	1.072	-0.41%	1.061
15/1/4	-12.63%	0.267	1.45%	0.842	2.26%	1.113	1.51%	1.088	1.14%	1.073
…...										
15/12/27	-2.85%	0.078	0.12%	0.862	0.90%	1.249	0.61%	1.194	0.46%	1.159
16/1/3	-19.66%	0.063	-14.73%	0.735	-10.44%	1.119	-6.95%	1.111	-5.20%	1.099
…...										
16/12/25	4.20%	0.131	5.87%	1.251	4.96%	1.554	3.32%	1.402	2.50%	1.318
17/1/1	1.15%	0.132	2.80%	1.286	3.28%	1.605	2.20%	1.432	1.66%	1.340
…...										
17/12/31	-10.88%	0.373	-0.59%	3.521	0.76%	3.392	0.52%	2.385	0.40%	1.976
18/1/7	2.83%	0.383	1.55%	3.576	-1.36%	3.346	-0.89%	2.363	-0.66%	1.962
…...										
18/12/23	1.00%	0.210	-1.71%	3.144	0.27%	3.227	0.19%	2.349	0.15%	1.971
18/12/30	5.12%	0.221	2.30%	3.216	1.34%	3.271	0.90%	2.370	0.69%	1.984

source: own elaboration

From above table, we can find when *k*=0, we only consider the maximization of return, so we only invest money into stock with highest expected return in the in-sample period. In the meanwhile, the final wealth is smallest and lower than the initial wealth 1, which meas we suffer a loss in this case. And the reason is same as in the backtesting of Markowitz model when *k*=0. When *k*=4, we have the highest final wealth.

The chart 4.10 shows the wealth evolution. The wealth under *k*=0 is the lowest in almost all the time. For each wealth evolution, they all increase in 2017 and begin to fall in 2018. The reasons are the bull market in Hong Kong in 2017 and bear market in 2018.

Chart 4.10 Wealth evolution



source: own elaboration

Then we need to calculate the performance measures of strategies with different small value of *k*, and the calculation procedure is same as the procedure in previous sub chapters. The results can be found in table 4.15.

Table 4.15 Results from different small value of *k*

	k=0	*k*=2	*k*=4	*k*=6	*k*=8
Final wealth (HKD)	0.221	3.216	3.271	2.370	1.984
Weekly portfolio return	-0.029%	0.598%	0.532%	0.365%	0.282%
Annual portfolio return	-1.488%	31.081%	27.667%	19.003%	14.671%
Weekly standard deviation	10.434%	5.469%	3.933%	2.622%	1.966%
Annual standard deviation	75.244%	39.437%	28.361%	18.907%	14.180%
Sharpe ratio	-0.006	0.103	0.127	0.127	0.127
Maximum drawdown	95.778%	41.318%	29.326%	20.114%	15.217%
Jensen's alpha	-3.413%	-1.789%	-1.096%	-0.731%	-0.548%

source: own elaboration

As we can see, when *k*=0, the values of Sharpe ratio and Jensen's alpha are lowest, and the value of maximum drawdown is the highest. So the strategy with *k*=0 is the worst among strategies in table 4.15. When *k*=4, 6, 8, we have the same value of Sharpe ratio, but when *k*=8, we have the lowest value of maximum drawdown. So we consider the strategy with *k*=8 as the best one.

4.4.3 Backtesting of Tobin Model with Large Value of *k*

In this section, we continue our backtesting with large value of *k*. When $k \to \infty$, the strategy is investment only into risk-free asset. The investment period is out-of-sample period (January 5th 2014-December 30th 2018). We use utility function to find the weights of stocks from in-sample period under large value of *k* and apply weights to out-of-sample period to get the weekly portfolio return and accumulated wealth in table 4.16. In this section, we don't title it as "backtesting of investment in risk-free asset" because we assume the large value of *k* is 99999, and according to our calculation, the weight of risk-free asset didn't reach 100%, but it's approaching to 100%. The weights of stocks when *k*=99999 in detail can also be found in annex.

Table 4.16 Portfolio returns and wealth under large value of *k*

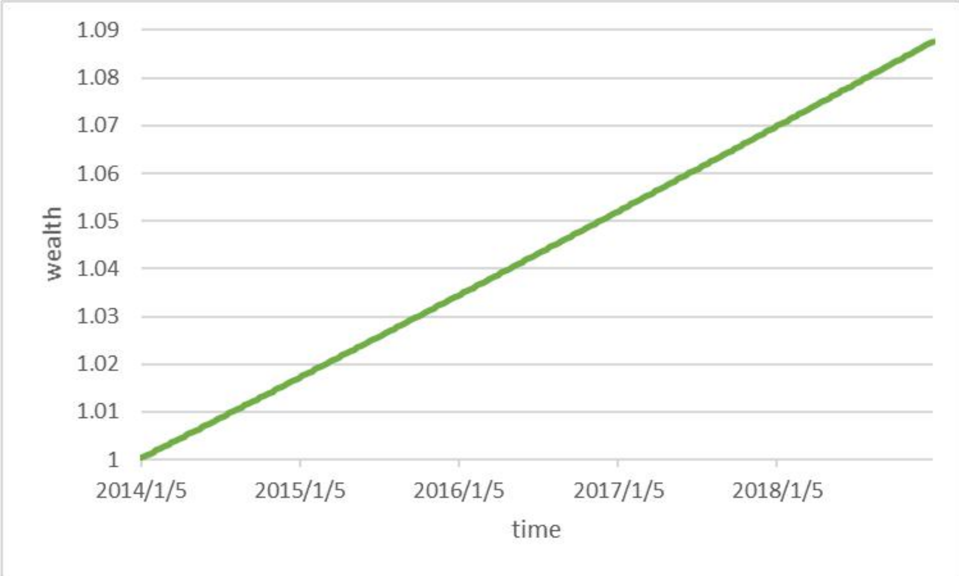
	k=99999			*k*=99999	
Date	returns	wealth	Date	returns	wealth
2014/1/5	0.03%	1.000	…...		
2014/1/12	0.03%	1.001	2016/12/25	0.03%	1.052
…...			2017/1/1	0.03%	1.052
2014/12/28	0.03%	1.017	…...		
2015/1/4	0.03%	1.017	2017/12/31	0.03%	1.070
…...			2018/1/7	0.03%	1.070
2015/12/27	0.03%	1.034	…...		
2016/1/3	0.03%	1.034	2018/12/23	0.03%	1.087
…...			2018/12/30	0.03%	1.088

source: own elaboration

From above table, we can find that when *k*=99999, the return is approaching the risk-free rate 0.0322%, because when $k \to \infty$, we only invest into the risk-free asset. The final wealth we get is 1.088 HKD, which means we earn 0.88 HKD after investment in out-of-sample period.

The wealth evolution when *k*=99999 in Tobin model is shown in chart 4.11. The trend of wealth is increasing slightly among this time period, because we nearly suffer no risk, by investing weekly, we get higher and higher wealth.

Chart 4.11 Wealth evolution



source: own elaboration

Next we need to calculate the performance measures of strategies with large value of *k*, and the calculation procedure is same as the procedure in previous sub chapters. The results can be found in table 4.17. It's worth noting that the maximum drawdown is 0%. This echoes the trend of wealth in chart 4.11.

Table 4.17 Results from large value of *k*

	k=99999
Final wealth (HKD)	1.088
Weekly portfolio return	0.032%
Annual portfolio return	1.675%
Weekly standard deviation	0.000%
Annual standard deviation	0.001%
Sharpe ratio	0.113
Maximum drawdown	0.000%
Jensen's alpha	0.000%

source: own elaboration

4.5 Comparison of Strategies

In this section, we are going to compare the strategies of portfolio optimization that we use. As we divide our data into in-sample and out-of-sample period, we backtest our strategies in out-of-sample period. The results from backtesting of each strategy are different, we compare the final wealth and performance measures to find the best strategy.

Firstly, we compare the final wealth we generate under each condition. Final wealth is how much we can get after investment in out-of-sample period with 1 HKD as initial wealth. We make a table to show the final wealth under each strategy, and we rank them from highest value to lowest, because investors always prefer higher wealth. In table 4.18, we can find that the final wealth of Naive strategy ranks 10th among 13 strategies, which is not good. Because in Naive strategy, we invest equally in each stock regardless of returns and risks of stocks. In Markowitz model and Tobin model, when k=0, we suffer a loss due to the great bias existed in the maximum return portfolio; when k=99999, we concentrate on investment with low risk, thus the final wealth are low. It's obviously that we get highest final wealth when k=2 under Markowitz model.

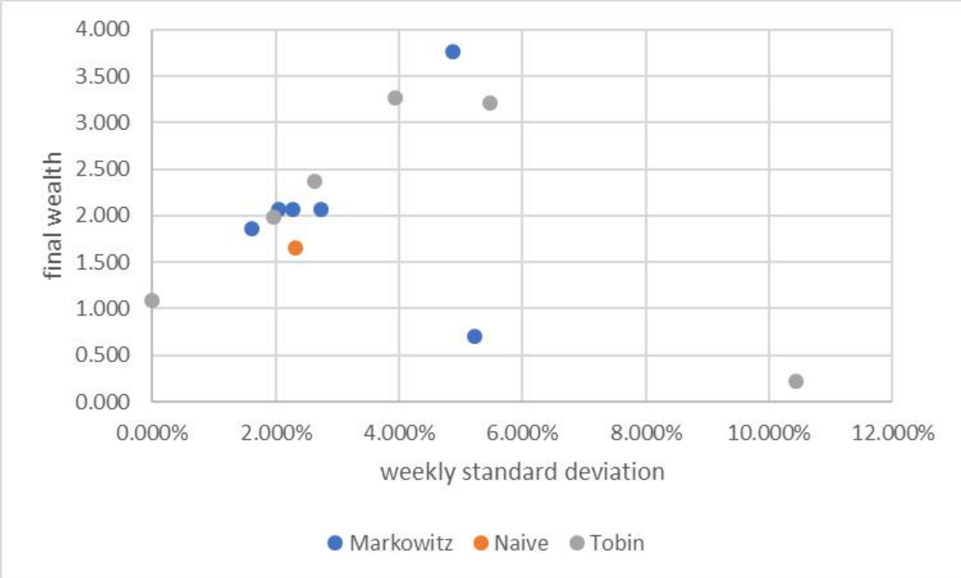
Table 4.18 Comparison of final wealth

Strategy		Final wealth	Rank
Naive strategy		1.652	10
Markowitz model	k=0	0.705	12
	k=2	3.761	1
	k=4	2.062	7
	k=6	2.073	5
	k=8	2.064	6
	k=99999	1.858	9
Tobin model	k=0	0.221	13
	k=2	3.216	3
	k=4	3.271	2
	k=6	2.370	4
	k=8	1.984	8
	k=99999	1.088	11

source: own elaboration

The following chart 4.12 shows the relationship between weekly standard deviation (risk) and final wealth. Most of the points follow the trend that the higher the standard deviation, the higher the final wealth. The point with highest standard deviation has the lowest final wealth and that's k=0 under Tobin model. Combined with table 4.18 and chart 4.12, we can find that when $k \in (0, \infty)$, with less risk aversion, we get higher wealth.

Chart 4.12 Final wealth and standard deviation



source: own elaboration

Then we compare the performance measures of each strategy. First, we compare different values of Sharpe ratio of each strategy in out-of-sample period. As we have mentioned in chapter 2, the Sharpe ratio represents that how many excess return an investor get for each additional risk. The higher the Sharpe ratio, the better the performance of strategy.

Table 4.19 Comparison of Sharpe ratio

Strategy		Sharpe ratio	Rank
Naive strategy		0.081	11
Markowitz model	*k*=0	-0.006	12
	k=2	0.122	6
	k=4	0.103	9
	k=6	0.120	7
	k=8	0.130	2
	k=99999	0.136	1
Tobin model	*k*=0	-0.006	12
	k=2	0.103	9
	k=4	0.127	3
	k=6	0.127	3
	k=8	0.127	3
	k=99999	0.113	8

source: own elaboration

Table 4.19 shows the comparison of values of Sharpe ratio under each strategy. We rank the strategies from highest value of Sharpe ratio to the lowest, which means the smaller the number of ranking, the higher the value of Sharpe ratio.

From the above table, we can find that the value of Sharpe ratio of Naive strategy ranks 11th among 13 strategies, which is similar to its ranking in final wealth. Both in Markowitz model and Tobin model, when k=0, the values of Sharpe ratio are same and negative, because they have low weekly portfolio returns and high weekly standard deviations. Moreover, when k=99999 under Markowitz model, the value of Sharpe ratio is the highest. When k=8 under Markowitz model, we get the second highest value of Sharpe ratio. And when k=4, 6, 8 under Tobin model, the values of Sharpe ratio are same. So we can select 5 relatively good strategies depending only on the value of Sharpe ratio.

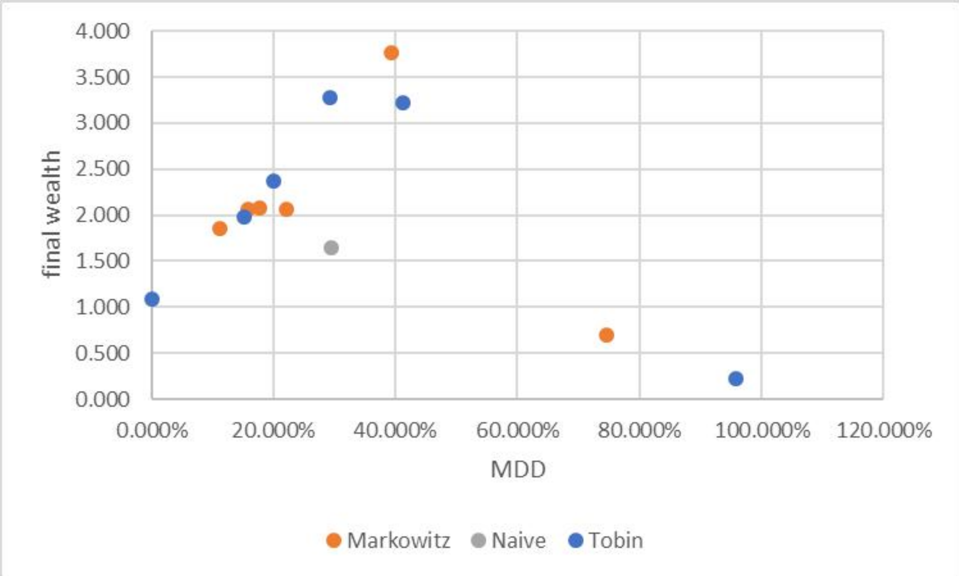
Next, we are going to compare the value of Maximum drawdown of each strategy. According to chapter 2, the lower the value of Maximum drawdown, the better the performance of strategy. So we rank the values of Maximum drawdown from lowest to highest. It means that the lower the number of ranking, the lower the value of maximum drawdown.

Table 4.20 Comparison of Maximum drawdown

Strategy		Maximum drawdown	Rank
Naive strategy		29.545%	9
Markowitz model	k=0	74.579%	12
	k=2	39.366%	10
	k=4	22.178%	7
	k=6	17.767%	5
	k=8	15.825%	4
	k=99999	11.180%	2
Tobin model	k=0	95.778%	13
	k=2	41.318%	11
	k=4	29.326%	8
	k=6	20.114%	6
	k=8	15.217%	3
	k=99999	0.000%	1

source: own elaboration

Chart 4.13 Maximum drawdown and wealth



source: own elaboration

There are two special points in the chart. The rightmost one is the point with highest maximum drawdown and lowest wealth, which is when *k*=0 under Tobin model. The point with second lowest wealth and second largest value of maximum drawdown is the strategy when *k*=0 under Markowitz model. We consider these two strategies as the worst strategies. For the other points, the general trend is that the higher the maximum drawdown, the higher the wealth. For example, in Naive strategy, the value of maximum drawdown ranks 9 and the wealth ranks 10. The point with lowest maximum drawdown has the third lowest final wealth, and it is the strategy when *k*=99999 under Tobin model. Both in Markowitz model and Tobin model, the higher the value of *k*, the lower the value of maximum drawdown. Because the higher the value of *k*, the deeper degree of risk aversion, the lower risk we suffer, the less volatile the wealth evolution is. According only to the rank in values of maximum drawdown, the Tobin model with *k*=99999 is the best strategy.

Then we compare the values of Jensen's alpha of each strategy. As we introduced in section 3.2.3, when the Jensen index is higher than 0, it means that the performance of the portfolio is better than the market benchmark portfolio. And the higher the excess part, the better the performance. The table 4.21 shows the value of Jensen's alpha under each strategy, and ranks them from largest to lowest.

Table 4.21 Comparison of Jensen's alpha

Strategy		Jensen's alpha	Rank
Naive strategy		-0.202%	3
Markowitz model	*k*=0	-1.707%	11
	k=2	-0.939%	9
	k=4	-0.894%	8
	k=6	-0.679%	6
	k=8	-0.552%	5
	k=99999	-0.037%	2
Tobin model	*k*=0	-3.413%	13
	k=2	-1.789%	12
	k=4	-1.096%	10
	k=6	-0.731%	7
	k=8	-0.548%	4
	k=99999	0.000%	1

source: own elaboration

According to table 4.21, the value of Jensen's alpha of Naive strategy ranks 3rd among all strategies. The strategy with highest value of Jensen's alpha is *k*=99999 under Tobin model and the strategy with second highest value of Jensen's alpha is *k*=99999 under Markowitz model. Both in Markowitz model and Tobin model, the higher the value of *k*, the higher the value of Jensen's alpha. Moreover, under each certain value of *k*, the value of Jensen's alpha under Tobin model is lower than Markowitz model. For example, when *k*=2, the Jensen's alpha under Tobin model ranks 12, but the Jensen's alpha under Markowitz model ranks 9. It means that in general, Markowitz model performs better than Tobin model under the measure of performance of Jensen's alpha.

We can make a table to show the complete results of all strategies, and it is put in annex. But in this section, we make a simple table to show the results of final wealth, weekly standard deviation and performance measures.

Table 4.22 Selected results of each Strategy

Strategy		Final wealth	Weekly standard deviation	Sharpe ratio	Maximum drawdown	Jensen's alpha
Naive strategy		1.652	2.315%	0.081	29.545%	-0.202%
Markowitz model	*k*=0	0.705	5.217%	-0.006	74.579%	-1.707%
	k=2	3.761	4.882%	0.122	39.366%	-0.939%
	k=4	2.062	2.734%	0.103	22.178%	-0.894%
	k=6	2.073	2.273%	0.120	17.767%	-0.679%
	k=8	2.064	2.055%	0.130	15.825%	-0.552%
	k=99999	1.858	1.603%	0.136	11.180%	-0.037%
Tobin model	*k*=0	0.221	10.434%	-0.006	95.778%	-3.413%
	k=2	3.216	5.469%	0.103	41.318%	-1.789%
	k=4	3.271	3.933%	0.127	29.326%	-1.096%
	k=6	2.370	2.622%	0.127	20.114%	-0.731%
	k=8	1.984	1.966%	0.127	15.217%	-0.548%
	k=99999	1.088	0.000%	0.113	0.000%	0.000%

source: own elaboration

In above table, the results of each column are ranked from best to worst. Expressed in colors, the order is green-yellow-red. It means that under each column, the greener cell represents better result.

Compared all the strategies, we can find that when *k*=99999 under Tobin model, the values of weekly standard deviation and maximum drawdown are lowest, and the value of Jensen's alpha is highest. But in this case, we almost only invest into risk-free assets, so the value of final wealth and Sharpe ratio are quite low compared to other strategy. We don't consider this strategy as the best strategy.

When *k*=0, it has the worst performance in each model respectively. For example, in Markowitz model, when *k*=0, it has lowest final wealth, highest standard deviation, lowest Sharpe ratio, highest maximum drawdown and lowest Jensen's alpha. And *k*=0 under Tobin model is the worst strategy among all 13 strategies. Under each certain value of *k*, the performances under Markowitz model and Tobin model are more or less similar.

We have 12 alternative strategies except *k*=99999 under Tobin model. It's obviously that *k*=99999 under Markowitz model has the best data in all three performance measures, that is, the highest Sharpe ratio, lowest maximum drawdown, highest Jensen's alpha. Moreover, it also has the lowest standard deviation. Although the final wealth is not the highest, compared to the initial wealth which is 1 HKD, the final wealth of k=99999 under Markowitz model is almost double of initial wealth, so the result is acceptable. Thus we consider the strategy as the best strategy.

5. Conclusion

In this thesis, we apply different portfolio allocation strategies to the historical data. The data are adjusted close prices of 30 stocks we select from the Hang Seng Index. They are weekly data between January 4th 2009 and December 30th 2018. We divide the data into two halves, half of which is in-sample period (January 4th 2009-December 29th 2013), and the other half is out-of-sample period (January 5th 2014-December 30th 2018).

The goal of the diploma thesis is to compare the out-of-sample performance of portfolio allocation strategies we select. The selected strategies are Naive strategy, Markowitz model, minimum variance portfolio and Tobin model.

In chapter 2, we introduce the portfolio theory, the historical data approach, the strategies we choose, and expected utility function which helps us to find the optimal portfolio. In chapter 3, we describe the backtesting framework and performance measures we choose, which are maximum drawdown, Sharpe ratio and Jensen's alpha.

In chapter 4, we apply naive strategy to out-of-sample period directly, because the weights of each stock are same. We apply Markowitz model and Tobin model in the in-sample period separately to construct the efficient frontier. Then in the backtesting procedure, according to different value of k, we use utility function to determine the weights of portfolios and apply them to out-of-sample period to get the portfolio returns and wealth under each strategy. When k=99999 under Markowitz model, the optimal portfolio is approaching the minimum variance portfolio. As we described in section 2.7, parameter k is the attitude of investor to risk, when $k \in (0, \infty)$, the lower the value of k, the less risk aversion the investor is.

After calculating the values of performance measures for each strategy, we compare the performance of strategies. Compared all the strategies, we can find that the strategy with k=99999 under Tobin model has the lowest values of weekly standard deviation and maximum drawdown, the highest value of Jensen's alpha, but also has the low value of final wealth and Sharpe ratio. And most importantly, we only invest into risk-free asset in this case, so we don't consider this strategy. When k=0, it has the worst performance in both Markowitz and Tobin model.

After removing k=99999 under Tobin model, we find that k=99999 under Markowitz model (minimum variance portfolio) has the best data in all three performance measures and it also has the lowest standard deviation and acceptable final wealth which is almost double of the initial wealth (1 HKD). Thus we consider the strategy as the best strategy.

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List of Abbreviations

σ_P	Standard deviation of the portfolio return
$\sigma_P{}^2$	Variance of the portfolio return
$\sigma_{i,j}$	Covariance of returns
DD	Drawdown
EMH	Efficient markets hypothesis
$E(R_P)$	Expected return of portfolio
HKD	Hong Kong dollar
HSI	Hang Seng Index
MDD	Maximum drawdown
Q	Covariance matrix
R_F	The risk-free rate
SR	Sharpe ratio
W_t	The accumulated wealth at time t
w_i	The weights of stock i
x_i	The weights of stock i

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List of Annexes

Annex 1

Weights of stocks under each strategy

Stocks	Naive strategy	Markowitz model						Stocks	Tobin model					
		k=0	*k*=2	*k*=4	*k*=6	*k*=8	*k*=99999		*k*=0	*k*=2	*k*=4	*k*=6	*k*=8	*k*=99999
TC	3.33%	0.00%	2.93%	14.03%	11.24%	9.42%	0.00%	TC	0.00%	28.10%	18.63%	12.42%	9.32%	0.0005%
ICBC	3.33%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	ICBC	0.00%	0.00%	0.00%	0.00%	0.00%	0.0001%
CCB	3.33%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	CCB	0.00%	0.00%	0.00%	0.00%	0.00%	0.0001%
PA	3.33%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	PA	0.00%	0.00%	0.00%	0.00%	0.00%	0.0001%
CM	3.33%	0.00%	0.00%	0.00%	0.00%	0.00%	4.25%	CM	0.00%	0.00%	0.00%	0.00%	0.00%	0.0000%
HSBC	3.33%	0.00%	0.00%	0.00%	0.00%	0.00%	0.23%	HSBC	0.00%	0.00%	0.00%	0.00%	0.00%	0.0000%
BCL	3.33%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	BCL	0.00%	0.00%	0.00%	0.00%	0.00%	0.0000%
CLI	3.33%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	CLI	0.00%	0.00%	0.00%	0.00%	0.00%	0.0000%
CPC	3.33%	0.00%	0.00%	0.00%	0.00%	0.00%	1.75%	CPC	0.00%	0.00%	0.00%	0.00%	0.00%	0.0001%
CNOOC	3.33%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	CNOOC	0.00%	0.00%	0.00%	0.00%	0.00%	0.0000%
BCC	3.33%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	BCC	0.00%	0.00%	0.00%	0.00%	0.00%	0.0000%
SHKP	3.33%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	SHKP	0.00%	0.00%	0.00%	0.00%	0.00%	0.0001%
CSE	3.33%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	CSE	0.00%	0.00%	0.00%	0.00%	0.00%	0.0001%
HSB	3.33%	0.00%	0.00%	0.00%	0.00%	0.00%	1.80%	HSB	0.00%	0.00%	0.00%	0.00%	0.00%	0.0001%
BOC	3.33%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	BOC	0.00%	0.00%	0.00%	0.00%	0.00%	0.0001%
CITIC	3.33%	0.00%	0.00%	0.00%	0.00%	0.00%	1.63%	CITIC	0.00%	0.00%	0.00%	0.00%	0.00%	0.0000%
HKEX	3.33%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	HKEX	0.00%	0.00%	0.00%	0.00%	0.00%	0.0001%
COLI	3.33%	0.00%	0.00%	0.00%	0.00%	0.00%	0.13%	COLI	0.00%	0.00%	0.00%	0.00%	0.00%	0.0000%
CKH	3.33%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	CKH	0.00%	0.00%	0.00%	0.00%	0.00%	0.0001%
MTR	3.33%	0.00%	0.00%	0.00%	0.00%	0.00%	1.57%	MTR	0.00%	0.00%	0.00%	0.00%	0.00%	0.0001%

CU	3.33%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	CU	0.00%	0.00%	0.00%	0.00%	0.00%	0.0001%
HKCG	3.33%	0.00%	0.00%	0.00%	0.00%	0.00%	1.83%	HKCG	0.00%	0.00%	0.00%	0.00%	0.00%	0.0001%
CG	3.33%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	CG	0.00%	0.00%	0.00%	0.00%	0.00%	0.0000%
GE	3.33%	100.00%	57.33%	32.79%	23.17%	18.10%	0.00%	GE	200.00%	65.59%	36.01%	24.01%	18.00%	0.0012%
CRL	3.33%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	CRL	0.00%	0.00%	0.00%	0.00%	0.00%	0.0000%
CLP	3.33%	0.00%	0.00%	0.00%	0.00%	5.65%	48.08%	CLP	0.00%	0.00%	3.35%	2.23%	1.67%	0.0002%
HLD	3.33%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	HLD	0.00%	0.00%	0.00%	0.00%	0.00%	0.0001%
LRE	3.33%	0.00%	0.00%	23.57%	29.18%	29.82%	17.17%	LRE	0.00%	47.18%	56.91%	37.94%	28.46%	0.0012%
CKI	3.33%	0.00%	0.00%	3.45%	17.80%	22.17%	19.22%	CKI	0.00%	6.79%	41.63%	27.75%	20.82%	0.0010%
SZ	3.33%	0.00%	39.73%	26.16%	18.61%	14.85%	2.33%	SZ	0.00%	52.34%	29.17%	19.45%	14.59%	0.0012%
sum	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	R_F	-100.00%	-100.00%	-85.70%	-23.80%	7.15%	99.9934%
								sum	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%

Annex 2

Complete results from each strategy

Strategy		Final wealth (HKD)	Weekly portfolio return	Annual portfolio return	Weekly standard deviation
Naive strategy		1.652	0.219%	11.408%	2.315%
Markowitz model	*k*=0	0.705	0.002%	0.093%	5.217%
	k=2	3.761	0.628%	32.636%	4.882%
	k=4	2.062	0.315%	16.370%	2.734%
	k=6	2.073	0.305%	15.885%	2.273%
	k=8	2.064	0.299%	15.555%	2.055%
	k=99999	1.858	0.250%	13.018%	1.603%
Tobin model	*k*=0	0.221	-0.029%	-1.488%	10.434%
	k=2	3.216	0.598%	31.081%	5.469%
	k=4	3.271	0.532%	27.667%	3.933%
	k=6	2.370	0.365%	19.003%	2.622%
	k=8	1.984	0.282%	14.671%	1.966%
	k=99999	1.088	0.032%	1.675%	0.000%
Strategy		**Annual standard deviation**	**Sharpe ratio**	**Maximum drawdown**	**Jensen's alpha**
Naive strategy		16.693%	0.081	29.545%	-0.202%
Markowitz model	*k*=0	37.622%	-0.006	74.579%	-1.707%
	k=2	35.205%	0.122	39.366%	-0.939%
	k=4	19.712%	0.103	22.178%	-0.894%
	k=6	16.392%	0.120	17.767%	-0.679%
	k=8	14.821%	0.130	15.825%	-0.552%
	k=99999	11.562%	0.136	11.180%	-0.037%
Tobin model	*k*=0	75.244%	-0.006	95.778%	-3.413%
	k=2	39.437%	0.103	41.318%	-1.789%
	k=4	28.361%	0.127	29.326%	-1.096%
	k=6	18.907%	0.127	20.114%	-0.731%
	k=8	14.180%	0.127	15.217%	-0.548%
	k=99999	0.001%	0.113	0.000%	0.000%

Annex 3

Covariance matrix of Markowitz model

	TC	ICBC	CCB	PA	CM	HSBC	BCL	CLI	CPC	CNOOC	BCC	SHKP	CSE	HSB	BOC
TC	0.223%	0.106%	0.088%	0.123%	0.043%	0.082%	0.093%	0.099%	0.063%	0.106%	0.107%	0.095%	0.106%	0.059%	0.068%
ICBC	0.106%	0.214%	0.159%	0.169%	0.065%	0.099%	0.157%	0.135%	0.106%	0.141%	0.174%	0.107%	0.136%	0.077%	0.110%
CCB	0.088%	0.159%	0.167%	0.141%	0.058%	0.095%	0.137%	0.115%	0.086%	0.113%	0.147%	0.089%	0.119%	0.070%	0.093%
PA	0.123%	0.169%	0.141%	0.280%	0.063%	0.098%	0.150%	0.194%	0.101%	0.160%	0.180%	0.122%	0.156%	0.091%	0.136%
CM	0.043%	0.065%	0.058%	0.063%	0.099%	0.064%	0.052%	0.063%	0.051%	0.082%	0.072%	0.051%	0.071%	0.048%	0.044%
HSBC	0.082%	0.099%	0.095%	0.098%	0.064%	0.189%	0.082%	0.088%	0.080%	0.115%	0.110%	0.087%	0.116%	0.086%	0.098%
BCL	0.093%	0.157%	0.137%	0.150%	0.052%	0.082%	0.179%	0.119%	0.076%	0.110%	0.153%	0.098%	0.117%	0.068%	0.101%
CLI	0.099%	0.135%	0.115%	0.194%	0.063%	0.088%	0.119%	0.215%	0.092%	0.131%	0.149%	0.093%	0.140%	0.064%	0.094%
CPC	0.063%	0.106%	0.086%	0.101%	0.051%	0.080%	0.076%	0.092%	0.160%	0.108%	0.114%	0.072%	0.108%	0.047%	0.072%
CNOOC	0.106%	0.141%	0.113%	0.160%	0.082%	0.115%	0.110%	0.131%	0.108%	0.234%	0.153%	0.119%	0.151%	0.087%	0.108%
BCC	0.107%	0.174%	0.147%	0.180%	0.072%	0.110%	0.153%	0.149%	0.114%	0.153%	0.217%	0.117%	0.164%	0.089%	0.118%
SHKP	0.095%	0.107%	0.089%	0.122%	0.051%	0.087%	0.098%	0.093%	0.072%	0.119%	0.117%	0.159%	0.113%	0.081%	0.100%
CSE	0.106%	0.136%	0.119%	0.156%	0.071%	0.116%	0.117%	0.140%	0.108%	0.151%	0.164%	0.113%	0.239%	0.081%	0.106%
HSB	0.059%	0.077%	0.070%	0.091%	0.048%	0.086%	0.068%	0.064%	0.047%	0.087%	0.089%	0.081%	0.081%	0.110%	0.086%
BOC	0.068%	0.110%	0.093%	0.136%	0.044%	0.098%	0.101%	0.094%	0.072%	0.108%	0.118%	0.100%	0.106%	0.086%	0.169%
CITIC	0.106%	0.142%	0.132%	0.184%	0.065%	0.134%	0.121%	0.152%	0.103%	0.149%	0.153%	0.123%	0.164%	0.095%	0.115%
HKEX	0.116%	0.158%	0.139%	0.184%	0.083%	0.136%	0.130%	0.147%	0.113%	0.167%	0.169%	0.129%	0.164%	0.109%	0.133%
COLI	0.115%	0.156%	0.131%	0.180%	0.046%	0.085%	0.140%	0.143%	0.065%	0.132%	0.150%	0.119%	0.133%	0.076%	0.093%
CKH	0.101%	0.113%	0.096%	0.127%	0.050%	0.090%	0.105%	0.100%	0.075%	0.117%	0.124%	0.131%	0.111%	0.085%	0.102%
MTR	0.061%	0.062%	0.058%	0.073%	0.033%	0.052%	0.067%	0.055%	0.044%	0.067%	0.074%	0.072%	0.067%	0.056%	0.063%
CU	0.074%	0.091%	0.068%	0.095%	0.058%	0.053%	0.065%	0.085%	0.064%	0.103%	0.099%	0.058%	0.102%	0.047%	0.049%

HKCG	0.040%	0.050%	0.039%	0.043%	0.023%	0.033%	0.047%	0.041%	0.035%	0.044%	0.058%	0.048%	0.051%	0.036%	0.034%
CG	0.136%	0.196%	0.166%	0.203%	0.071%	0.123%	0.157%	0.151%	0.111%	0.168%	0.183%	0.139%	0.169%	0.097%	0.134%
GE	0.142%	0.136%	0.120%	0.171%	0.042%	0.135%	0.127%	0.128%	0.090%	0.126%	0.138%	0.113%	0.140%	0.083%	0.113%
CRL	0.109%	0.161%	0.137%	0.188%	0.042%	0.090%	0.160%	0.151%	0.079%	0.128%	0.154%	0.136%	0.127%	0.078%	0.110%
CLP	0.011%	0.011%	0.013%	0.015%	0.012%	0.013%	0.013%	0.011%	0.012%	0.016%	0.016%	0.013%	0.014%	0.013%	0.014%
HLD	0.099%	0.121%	0.105%	0.140%	0.059%	0.100%	0.113%	0.102%	0.088%	0.120%	0.134%	0.149%	0.128%	0.097%	0.116%
LRE	0.017%	0.026%	0.025%	0.026%	0.018%	0.012%	0.024%	0.020%	0.012%	0.025%	0.031%	0.023%	0.017%	0.022%	0.025%
CKI	0.013%	-0.002%	-0.005%	-0.005%	0.008%	0.001%	0.003%	0.000%	0.005%	0.006%	0.001%	0.004%	-0.001%	0.000%	-0.005%
SZ	0.078%	0.091%	0.068%	0.075%	0.036%	0.061%	0.055%	0.071%	0.070%	0.082%	0.088%	0.075%	0.089%	0.052%	0.050%
	CITIC	HKEX	COLI	CKH	MTR	CU	HKCG	CG	GE	CRL	CLP	HLD	LRE	CKI	SZ
TC	0.106%	0.116%	0.115%	0.101%	0.061%	0.074%	0.040%	0.136%	0.142%	0.109%	0.011%	0.099%	0.017%	0.013%	0.078%
ICBC	0.142%	0.158%	0.156%	0.113%	0.062%	0.091%	0.050%	0.196%	0.136%	0.161%	0.011%	0.121%	0.026%	-0.002%	0.091%
CCB	0.132%	0.139%	0.131%	0.096%	0.058%	0.068%	0.039%	0.166%	0.120%	0.137%	0.013%	0.105%	0.025%	-0.005%	0.068%
PA	0.184%	0.184%	0.180%	0.127%	0.073%	0.095%	0.043%	0.203%	0.171%	0.188%	0.015%	0.140%	0.026%	-0.005%	0.075%
CM	0.065%	0.083%	0.046%	0.050%	0.033%	0.058%	0.023%	0.071%	0.042%	0.042%	0.012%	0.059%	0.018%	0.008%	0.036%
HSBC	0.134%	0.136%	0.085%	0.090%	0.052%	0.053%	0.033%	0.123%	0.135%	0.090%	0.013%	0.100%	0.012%	0.001%	0.061%
BCL	0.121%	0.130%	0.140%	0.105%	0.067%	0.065%	0.047%	0.157%	0.127%	0.160%	0.013%	0.113%	0.024%	0.003%	0.055%
CLI	0.152%	0.147%	0.143%	0.100%	0.055%	0.085%	0.041%	0.151%	0.128%	0.151%	0.011%	0.102%	0.020%	0.000%	0.071%
CPC	0.103%	0.113%	0.065%	0.075%	0.044%	0.064%	0.035%	0.111%	0.090%	0.079%	0.012%	0.088%	0.012%	0.005%	0.070%
CNOOC	0.149%	0.167%	0.132%	0.117%	0.067%	0.103%	0.044%	0.168%	0.126%	0.128%	0.016%	0.120%	0.025%	0.006%	0.082%
BCC	0.153%	0.169%	0.150%	0.124%	0.074%	0.099%	0.058%	0.183%	0.138%	0.154%	0.016%	0.134%	0.031%	0.001%	0.088%
SHKP	0.123%	0.129%	0.119%	0.131%	0.072%	0.058%	0.048%	0.139%	0.113%	0.136%	0.013%	0.149%	0.023%	0.004%	0.075%
CSE	0.164%	0.164%	0.133%	0.111%	0.067%	0.102%	0.051%	0.169%	0.140%	0.127%	0.014%	0.128%	0.017%	-0.001%	0.089%
HSB	0.095%	0.109%	0.076%	0.085%	0.056%	0.047%	0.036%	0.097%	0.083%	0.078%	0.013%	0.097%	0.022%	0.000%	0.052%

BOC	0.115%	0.133%	0.093%	0.102%	0.063%	0.049%	0.034%	0.134%	0.113%	0.110%	0.014%	0.116%	0.025%	-0.005%	0.050%
CITIC	0.337%	0.180%	0.144%	0.120%	0.082%	0.095%	0.039%	0.206%	0.160%	0.163%	0.006%	0.133%	0.008%	-0.008%	0.103%
HKEX	0.180%	0.239%	0.143%	0.136%	0.080%	0.093%	0.047%	0.195%	0.164%	0.145%	0.014%	0.149%	0.034%	-0.007%	0.105%
COLI	0.144%	0.143%	0.316%	0.130%	0.071%	0.081%	0.044%	0.260%	0.173%	0.299%	0.008%	0.130%	0.027%	0.001%	0.064%
CKH	0.120%	0.136%	0.130%	0.153%	0.076%	0.067%	0.047%	0.138%	0.134%	0.138%	0.018%	0.143%	0.032%	0.009%	0.075%
MTR	0.082%	0.080%	0.071%	0.076%	0.074%	0.035%	0.033%	0.082%	0.067%	0.082%	0.016%	0.081%	0.015%	0.009%	0.051%
CU	0.095%	0.093%	0.081%	0.067%	0.035%	0.220%	0.041%	0.096%	0.053%	0.056%	0.015%	0.070%	0.024%	0.007%	0.034%
HKCG	0.039%	0.047%	0.044%	0.047%	0.033%	0.041%	0.068%	0.044%	0.045%	0.048%	0.020%	0.055%	0.016%	0.009%	0.030%
CG	0.206%	0.195%	0.260%	0.138%	0.082%	0.096%	0.044%	0.441%	0.186%	0.288%	0.003%	0.156%	0.032%	-0.005%	0.116%
GE	0.160%	0.164%	0.173%	0.134%	0.067%	0.053%	0.045%	0.186%	0.409%	0.190%	0.011%	0.132%	0.012%	0.005%	0.092%
CRL	0.163%	0.145%	0.299%	0.138%	0.082%	0.056%	0.048%	0.288%	0.190%	0.387%	0.008%	0.149%	0.021%	0.005%	0.077%
CLP	0.006%	0.014%	0.008%	0.018%	0.016%	0.015%	0.020%	0.003%	0.011%	0.008%	0.030%	0.018%	0.007%	0.011%	-0.003%
HLD	0.133%	0.149%	0.130%	0.143%	0.081%	0.070%	0.055%	0.156%	0.132%	0.149%	0.018%	0.202%	0.029%	-0.003%	0.067%
LRE	0.008%	0.034%	0.027%	0.032%	0.015%	0.024%	0.016%	0.032%	0.012%	0.021%	0.007%	0.029%	0.070%	0.007%	0.030%
CKI	-0.008%	-0.007%	0.001%	0.009%	0.009%	0.007%	0.009%	-0.005%	0.005%	0.005%	0.011%	-0.003%	0.007%	0.065%	0.003%
SZ	0.103%	0.105%	0.064%	0.075%	0.051%	0.034%	0.030%	0.116%	0.092%	0.077%	-0.003%	0.067%	0.030%	0.003%	0.359%